

The General Impossibility of Neoclassical Economics: Or Does Bertrand Russell Deserve a Nobel Prize for Economics?¹

1 Introduction

Today, mainstream economics prides itself on its mathematical rigour and deploys mathematics to an enormous extent as indicative of disciplinary acceptability, thereby policing the exclusion of other forms of economics to an extraordinary degree. How this situation came about, and with what consequences, are addressed in the next section, highlighting the extent to which the use of mathematics has promoted a particular content within economics, one that has shifted only in its expanding scope of application since the formalist revolution of the middle of the last century. This sets the context for the main goal of this contribution, to assess the extent to which formal problems within, and not of, mathematical reasoning itself set constraints on what can be achieved within mainstream economics. In particular, mathematics has found it necessary to negotiate the consequences of Russell's paradoxes, laid out in Section 3. Placing mathematics on sound foundations is found to have potential implications for, or limitations on, what can be achieved with mathematics in its applications. In particular, as argued in Section 4, for an economic theory based on methodological individualism, there are severe limitations upon the extent to which social properties can be consistently addressed – whether micro can be legitimately extrapolated to macro. In this light, the concluding remarks indulgently reflect on the likely continuing neglect of the implications for economic theory of Russell's paradoxes despite our best efforts.

2 Formalism and Content in Economic Theory

Mark Blaug (1999, 2001 and 2003) has dubbed the period between 1945 and 1955 as one in which economics went through a “formalist revolution”. There is no doubt that this decade does mark a watershed in the evolution of the discipline, and it is readily recognisable that use of mathematical presentation (and statistical techniques) ratcheted up in that period. It also gathered such a momentum that formal mathematical presentation is now taken as essential in what are perceived by orthodoxy to be all of the leading journals. As a major player in that formalist revolution, Debreu's (1986, p. 1265) commitment to formalism with economics, in a sense, as a conceptual add on, could not be plainer:

An axiomatized theory first selects its primitive concepts and represents each one of them by a mathematical object ... Next assumptions on the objects representing the primitive concepts are specified, and consequences are mathematically derived from them. The economic interpretation of the theorems so obtained is the last step of the analysis. According to the schema, an axiomatized theory has a mathematical form that is completely separated from its economic content. If one removes the economic interpretation of the primitive concepts, of the assumptions ... its bare mathematical structure must still stand.

But, as argued in Fine (2007b), the formalist revolution not only signified a turning point in the form taken by economics but also in its content and, especially, in its scope. From the marginalist revolution of the 1870s to the formalist revolution, the thrust of mathematical economics was to extract as much juice out of the idea of the optimising (supposedly “rational”) individuals as possible in the context of supply and demand for the market. This entailed an almost exclusive reliance upon calculus as far as technique is concerned, still much the same today, and the desire to obtain results took precedence over more or less everything else. This is true of the notion of rationality itself, reduced to maximisation of utility or, even more narrowly, a logic of choice over bundles of goods. Other motives and aspects of individuality, let alone social influence over interdependent preferences, were set aside as an inconvenience. Even for Robbins (1932, p. 87), notorious for his definition of economics as the allocation of scarce resources between competing ends, “economic subjects can be pure egoists, pure altruists, pure ascetics, pure sensualists or – what is much more likely – mixed bundles of all of these impulses”. There is a fair bit of individual character to ponder in this but it was soon to be discarded. Yet as Waller (2004, p. 1112) puts it in review of Davis (2003), “But if choices are the only characteristics of atomistic individuals, the theory of the individual becomes so reductionist that it ceases to be about human beings”. For Davis (2007, p. 203) himself, “if the basis of the atomistic individual was its inner life, and that inner life is now black-boxed into non-existence, then it follows that this neoclassical individual also ceases to exist”. In addition, as emphasised by Hutchison (1998)

around the time and by Lawson (1997) most recently, economic methodology was reduced to deductivism from these reduced axioms of individual economic behaviour.

Prior to the formalist revolution, the process of establishing these mathematical techniques involved what might be described as an implosion as the explanatory content and scope of application was diminished to the narrowest determinants of supply and demand. This left enormous scope for alternative forms of economics to prosper, not least Keynesian macroeconomics but also a whole range of approaches that would now be dubbed heterodox, then to a degree the orthodoxy of their time although now marginalised within the discipline. In addition, just to establish the technical apparatus associated with the optimising individual, essentially the now ubiquitous utility and production functions, the rise of mathematical methods within economics inevitably withdrew content from, and contact with, the other social sciences whose methods, theories and variables would have undermined the rationality enterprise.

Following the formalist revolution, the scope of application of the technical apparatus began a process of expansions. From a logical point of view, its principles, such as utility maximisation, are universal. So, once established in form and content as lying at the heart of the discipline, it was inevitable that the traditional confinement to (aggregate) supply and demand on the market should be breached. There began a process of colonising the discipline, so much so that Lucas (1987, p. 108) could claim, “the term ‘macroeconomic’ will simply disappear from use and the modifier ‘micro’ will be superfluous”, cited by Davis (2003, p. 35). And the forward march of microeconomic principles was also extended to other disciplines as the optimising individual became perceived as engaged in pursuit of self-interest across all activities, economic or otherwise. As argued by Fine and Milonakis (2008), such economics imperialism has gone through two phases. The first, with leading representative Gary Becker, perceived all economic and social life as if the consequence of optimising behaviour and as if perfectly markets were present. The second phase still draws upon the idea of economic rationality but with the non-market as the rational response to market imperfections.²

In short, economics created a mathematical technical apparatus in order to solve a particular problem of its own making, embracing huge qualifications along the way. These have now been conveniently forgotten in applying the technical apparatus as widely as possible. How, and with what confidence, this is now done is strikingly revealed by Demsetz (1997, p. 1). He opens, “The strong export surplus economics maintains in its trade in ideas and methods with the other social sciences is an important indicator of the success of economics. Not much has been said about the source of this success, but it has been attributed largely to advantages offered to other social sciences by the economics tool kit ... The emphasis here is on the broad scope of the phenomena that can be explained by our tool kit”, emphases added. Further, he continues by boasting of the achievements of the discipline, “Economics may be judged the more successful social science because it has explained phenomena within its traditional boundaries better than the other social sciences have explained phenomena within their respective traditional boundaries. The primacy of economics may be established in this sense even if economics never influenced the other social sciences”, p. 2.

Such sleight of hand in using the economist’s hammer to slice, as it were, the social scientist’s bread would be impossible but for the orthodox economist’s failure to respect knowledge of methodology and history of their own discipline, and to question a false stylised belief that their own practice emulates that of the natural sciences. Thus, significantly, in his defence of orthodoxy, Dasgupta (2002, p. 57) opens by confessing that, “Most economists ... have little time for the philosophy of economics as an intellectual discipline. They have even less patience with economic methodology. They prefer instead to *do* economics ... There is much to be said for this habit ... I know of no contemporary practicing economist whose investigations have been aided by the writings of professional methodologists”. Further, neglect of history of economic thought is justified by reference to the methods of the natural sciences, “You can emerge from your graduate studies in economics without having read any of the classics, or indeed, without having anything other than a vague notion of what the great thinkers of the past had written”, for “She reads Ricardo no more than the contemporary physicist reads James Clerk Maxwell”, p. 61.³

The corresponding drive to mathematical deductivism in principle, and its particular content in practice, can be seen to be a reflection of one side of Polanyi’s double movement in the rise of capitalism, disembedding homo economicus from society. Significantly, it is the rise of capitalism that prompted social theorists from Weber through Marshall, Schumpeter, Parsons and Polanyi to accept the

creation of an alienated economic self-interest as a social driving force, appropriately to be isolated and studied on the basis of methodological individualism (in contrast, for example, to the evolutionary and institutional schools of economic thought associated with Veblen). But each, in a different way and to a different extent, denied that such economic analysis could exhaust economics let alone social science. For Parsons, for example, sociology was to be distinguished from economics by its method and not by its subject matter. And with the subsequent rise of economics imperialism following the formalist revolution within economics, Polanyi's double movement is unconsciously parodied by re-embedding the social within the economic. In addition, the isolated project of extracting the technical implications of economic rationality was wedded to the more widespread emergence of modernism and analytical philosophy, in which as much thought and reason as possible is reduced to formal mathematics, thereby creating a dualism between the rational and the irrational (the artistic and the cultural for example). But, as a precondition, the same principle applied to mathematics itself. Could it be reduced to a coherent axiomatic and deductive system as the basis for its application to the natural or social worlds? Such was the problem posed for analytical philosophy in general and for the philosophical foundations of mathematics in particular, with Bertrand Russell to the fore.

3 Russell's Paradoxes

In 1950, Bertrand Russell was awarded the Nobel Prize for Literature. He might more appropriately have been honoured with the Prize for Peace, given his ardent commitment to the Campaign for Nuclear Disarmament. Yet, intellectually, his greatest contribution has been in philosophy for which there is no Nobel Prize. In particular, he is remembered for his paradoxes and their implications for the logical foundations of mathematics.

The purpose of this contribution is to examine the implications of Russell's paradoxes for economic theory. It has, after all, become highly mathematised. And, not surprisingly, there has been considerable debate over the appropriate contribution that mathematics can make to economic theory, especially from those adopting an increasingly marginalised heterodox position.⁴ Criticism of mathematical methods has been heavily concerned with whether their application to economics imposes limitations on the scope and content of theory or whether it is, or can be, purely a facilitating device for clarity and rigour. For Marshall, followed by Keynes, for example, mathematics served at most as an aid to clear thinking and not as a model or representation of the real world. Subsequently, focus of debate has been on the axiomatics and formalism of mainstream economics, as realised in its reliance upon the deductive method, together with its increasing reliance upon mathematical modes of rhetoric and a particular set of technical assumptions, Backhouse (1998). By complete contrast, attention here is focused on the almost neglected issue of the inner limitations of mathematics itself as a deductive method, prior to its application to economics, and the implications of these intrinsic limitations for economic theory.

This is where Russell's paradoxes are relevant.⁵ For, whatever the merits of mathematics in its application to economics, these are pre-conditioned by the limits of mathematics itself as a logical system, a matter that has tended to be entirely overlooked in the (methodology of) economics literature.⁶ Thus, there are problems, or limitations, within as well as of deductivism even as a mathematical method.

The most popular form in which Russell's paradoxes are known is through the following proposition: "the ship's barber shaves all those on board who do not shave themselves". The conundrum is that if the barber shaves himself, it follows that he does not, and vice-versa. The proposition would appear to be self-contradictory.

It is now generally accepted that this paradox is mere word play, a trick of language. The paradox can be resolved by denying the existence of such a barber, just as we can easily construct an inconsistency by assuming both A and not A. But Russell's paradoxes in set-theoretic terms are more challenging. Let S be the set of sets that do not belong to themselves. Then if S does belong to itself, it does not, and vice-versa. Inconsistency arises once more. To resolve it barber-wise would be to require that the set S does not exist, not a matter of semantics but a genuine limitation on our freedom to define the existence of sets.

One way of interpreting this conundrum, standard within mathematical logic, is in terms of an incompatibility between two different ways of defining sets. Sets can be defined in terms of their

properties or sets can be defined by the individual elements they contain – greenness, for example, as a property, as opposed to collecting objects together that happen to be green. As each of Russell’s paradoxes demonstrates, these two approaches are incompatible. We can define whom the barber shaves in terms of individuals or in terms of (not) shaving but not both. And we can define sets by their members or by their properties (belonging to or not) but not both without limit for risk of inconsistency.

Thus, an immediate way out of the paradoxes is to separate the elemental (what belongs or membership) notion of set from the relational (what property or class) notion. In other words, the paradoxes arise because, contrary to our intuition that these two approaches might be compatible, it is not possible to lump them together as a mutually consistent way of defining sets. As Gödel (1983, p. 452) puts it, Russell has the effect of “bringing to light the amazing fact that our logical intuitions (i.e., intuitions concerning such notions as: truth, concept, being, class, etc.) are self-contradictory”. In short, mathematics itself does not allow us a free hand in the formation of “sets”, by free use of both elemental membership and relational property.

This, then, raises the question of exactly how far we can go in extending the elemental/individual/membership definition of sets to incorporate the relational/properties/class definition without re-introducing Russell’s paradoxes, or vice-versa. Gödel points out that Russell saw resolution in “two possible directions ... which he called the zig-zag theory and the theory of limitations of size, respectively, and which might perhaps more significantly be called the intensional and the extensional theory. The second one would make the existence of a class or concept depend on the extension of the propositional function (requiring that it be not too big), the first one on its content or meaning (requiring a certain kind of ‘simplicity’, the precise formulation of which would be the problem)”, p. 452/3. Put more simply, we do not allow all sets to be defined whether by elements or properties without restriction. This might be done by excluding sets that belong to sets and so consideration of those that do not belong. Thus, “The paradoxes are avoided by the theory of simple types”, clarified in a footnote as, “individuals, properties of individuals, relations between individuals, properties of such relations, etc ... Mixed types ... are excluded”, emphasis added. He continues, “That the theory of simple types suffices for avoiding also the epistemological paradoxes is shown by a closer analysis of these”, p. 455. A weaker solution, due to Zermelo, would be that, “the sets are split up into ‘levels’ in such a manner that only sets of lower levels can be elements of sets of higher levels”, p. 459.

This way of proceeding has become known as the iterative (or extensional) concept of set as opposed to the intensional. As Wang (1983, p. 537) suggests:

The iterative concept of set is of course quite different from the dichotomy concept which regards each set as obtained by dividing the totality of all things into two categories (viz. those which have the property and those which do not).

In this light, Wang (1983, p. 541) identifies two different responses to the paradoxes, the “bankruptcy (contradiction) or misunderstanding (error)” theories. For the error approach, sets are perceived not to have been properly understood. Hence, the goal is to “uncover flaws in seemingly correct arguments” by restricting the definition of a set more in conformity to intuition, as for Zermelo and the iterative method, p. 542. For the bankrupts, basic intuition is contradictory and can only be salvaged by ad hoc devices such as Gödel’s appeal to Russell’s notion of simple types, where mathematical requirements or, more exactly, restrictions are imposed to preclude paradoxes. For Wang, in interpreting Gödel, there is a shift in position from misunderstanding/error to bankruptcy/contradiction corresponding to the shift from the foundations of mathematics to the more general logic of truth, concept, being, class, etc. Mathematics as such requires something like the iterative conception of sets in order to preclude contradictions but without thereby restricting the mathematical properties that are sought. But to impose such restrictions outside mathematics to a more general subject matter would be arbitrary. Indeed, “The full concept of class (truth, concept, being, etc.) is not used in mathematics, and the iterative concept, which is sufficient for mathematics, may or may not be the full concept of class ... In relation to logic as opposed to mathematics, Gödel believes that the unsolved difficulties are mainly in connection with the intensional paradoxes (such as the concept of not applying to itself) ... In terms of the contrast between bankruptcy and misunderstanding ... Gödel’s view is that the paradoxes in mathematics, which he identifies with set theory, are due to a misunderstanding, while logic, as far as its true principles are concerned, is bankrupt on account of the intensional paradoxes”, p. 537/8.

Now Benacerraf and Putnam (1983, p. 4) adopt a more tempered tone than the “bankruptcy (contradiction) or misunderstanding (error)” approach of Wang referring, respectively, to reformists and apologists. For them, “Inevitably, the reformers and the apologists rub elbows. But the distinction is a vague one and we should not try to make too much of it”. From a formal point of view, both sides need to offer restrictions on set formation in order to avoid contradiction. The issue is whether such restrictions are functional (allowing mathematics without in-built inconsistency), intuitive (apparently reasonable), or appropriate (to the subject matter other than mathematics but to which the mathematics is being applied). But what is clear is that the extensional, iterative (individualistic) approach is only at most demonstrably acceptable for the pure science of mathematics. Once it is extended to other areas, such as truth, concept, being, class, etc, the restrictions needed to provide the foundations for mathematics have neither intuition nor basis. As Boolos (1983, p. 490) puts it, citing Russell (1959, p. 80):

These theories appear to lack motivation that is independent of the paradoxes in the following sense: they are not, as Russell has written, “such as even the cleverest logician would have thought of if he had not known of the contradictions”. [Thus] a final and satisfying resolution to the set-theoretical paradoxes cannot be embodied in a theory that blocks their derivation by artificial technical restrictions on the set of axioms that are imposed *only because* paradox would otherwise ensue; these theories survive only through such artificial devices.

Put another way, mathematics can be rescued from the paradoxes by the iterative concept of set. But, there are other ways of resolving the paradoxes, those giving emphasis to the intensional as opposed to the extensional approach for example, each with its own properties and restrictions, Fine, K. (2006). But, in applying mathematics outside of its own immediate domain, the restrictions needed to avoid paradoxes should be specific and appropriate to the object of analysis involved. In a nutshell, underpinning consistency to ensure that $2+2=4$ is not the same thing as dealing with the conceptual foundations of the natural or the social sciences.

4 Implications for Economics

But it is precisely the application of mathematics to economic theory that is under scrutiny. Does it make sense to limit the formation of sets to some form of iterative concept in the context of economic theory? Or is some other resolution required of Russell’s paradoxes? Consider the new institutional economics. In a useful, if necessarily partial, overview of the evolution of the new institutional economics and a survey of its current state of play and prospects, Richter (2005, p. 171), himself the longstanding editor of the Journal of Institutional and Theoretical Economics, appropriately suggests that it is primarily based upon mainstream neo-classical economics:

the foundation stones of the NIE are the same as those of neoclassical economics: methodological individualism and individual rational choice given a set of constraints. However, due to transaction or information costs, information is limited and thus institutions matter.

As will be familiar to all academic economists, this means that the NIE accepts the importance of institutions but seeks to explain them, to endogenise them, albeit on the basis of aggregating over the optimising behaviour of individuals, this behaviour itself potentially modified in light of informational constraints and historically evolved institutions (and possibly other exogenously given motivations and limits on calculation). In short, there is a presumption that something, the institutional, exists independent of, but connected to, individuals. The latter both form the institutional and respond to it.

In this respect, there is a sharp difference with the old institutional economics (and the vast majority of non-rational choice social science) for which the institutional, and the social more generally, make up the analytical starting point. Not surprisingly, mainstream economics has displayed scant respect for such alternatives but has increasingly sought to reconstruct it on the basis of its own methodology, such is the nature of the current phase of “economics imperialism”, Field (1979) for an early recognition of this and Fine and Milonakis (2008) for a full discussion. Of course, the new institutional economics displays much more in motivation and content than relying exclusively upon rational choice. As such, it adopts mixed methods around individual and social behaviour. This still leaves open whether these mixes are mutually consistent when fully explored for their logical implications.

Interestingly, if necessarily inadvertently, Benacerraf and Putnam (1983, p. 28) pinpoint precisely the way in which the iterative concept of set parallels that of methodological individualism because, for the former, “there is *some* relation of ‘priority’ that is transitive, irreflexive, and asymmetrical, and such that the members of any set are always *prior* to the set”.⁷ That the extensional, in set theory, should prevail over the intensional, has its counterpart in methodological individualism – the elemental individual has priority over the social relational, structural or whatever. In other words, the institutional derives from the priority of the individual over the higher level without total feedback (reflex and symmetry).⁸ If the latter are allowed, paradoxes are liable to ensue. Is it possible, however, to formalise this correspondence between the foundations of set theory and (the new) institutional economics?

At first blush, even a casual perusal of the literature will reveal that there is ambiguity over the definition of what constitutes an institution, in the NIE and more broadly. The reason for this is relatively simple as it is an immediate consequence of an approach based on methodological individualism. Institution becomes a metaphor for all non-individualistic aspects, so it ranges over everything from collective action to ideology. But, putting this aside for the moment, an institution must, at the very least, involve more than one person if only in the limited sense of an individual at least reacting with something, if not necessarily somebody, else. Institutions can, for example, be impersonal, and relate to each other.

For the sake of simplicity for the moment, assume that society consists of just two people, *a* and *b*. In terms of forming an institution, whether it be mutual respect of property rights or common language or custom, then there is, in the first instance, only one option. This is the institution made up out of membership by both *a* and *b*. This is readily expressed in set-theoretic terms by $\{a, b\}$. Subject to the content of the institution, left unspecified for generality, does this fill out the institutional structure of this elementary society as there are no other sets of individuals other than the individuals themselves $\{a\}$ and $\{b\}$?

The answer is no and the reason for this is that both *a* and *b*, as individuals, can relate to the institution $\{a, b\}$. This might appear to be fanciful. Why not consider *a*’s or *b*’s relations to $\{a, b\}$ to be part and parcel of the institution of $\{a, b\}$ itself? First, there are logical reasons why not – the set $\{a, b\}$ is distinct from the set $\{a, \{a, b\}\}$, or $\{b, \{a, b\}\}$ and $\{a, b, \{a, b\}\}$ for that matter. More intuitively, $\{a, b\}$ is an institution to which its constituent members *a* and *b* can relate. If the institution $\{a, b\}$ is the one of language for example, *a* and *b* do not simply talk to one another, they also relate to the language itself independently of talking to one another. Otherwise, of course, the language could never change, as for it to do so depends upon *a* for example seeking to change $\{a, b\}$ – or, more exactly, the (here unspecified) institutional content attached to $\{a, b\}$. And the same applies whatever is the institutional substance attached to $\{a, b\}$, custom, property rights, ideology, and so on. In any case, more generally than for our two-person society, institutions, *I* and *J* say, also interact with one another to form the higher level institution, $\{I, J\}$ which, in membership terms, would be made up of a set of sets.

Thus, it would be an impoverished notion of institutions, and of institutional structure as a whole, if the only institutions considered are collections of individuals.⁹ For Douglass North, for example, one theory of the state is that it may represent the interests of the monarchy but the institutional structure depends upon how the governed customarily respond to corresponding property rights, ultimately rebelling against them when incentives to do so are strong enough, Fine and Milonakis (2003 and 2008) for a critical exposition. If the state and its citizens did not together constitute a higher level institution, the one could never be overthrown by the other whether for reasons of allocative efficiency, shifting property rights, or ideology.

Somewhat differently, Stiglitz (1989, p. 21) asks, “in what does the government have a comparative advantage?”, and answers in terms of its distinctiveness as an institution, namely that membership is universal and that the state has powers of compulsion, presumably over its citizens at a higher level than their membership of it alone. In other words, not only is the state embroiled institutionally with its citizens (and, presumably, other lower level institutions), it also has a relationship (of compulsion) to them by parallel with the intensional notion of set. In this case, the institutional structure includes the state and its citizens and the powers of compulsion. This is, in Gödel’s term a mixed type and should be ringing alarm bells. But such mixed types are common within

institutional economics as theory, and empirics, moves seamlessly between individual members and their behaviour and properties connecting them. In Peck's (2005) terms, it is all a bit "dirty".¹⁰ Nonetheless, once again, it is apparent that the literature does, absolutely correctly, perceive institutions in formal terms as embodying a complex set-theoretic content and not simply sets of individuals.

In this respect, Akerlof's work is particularly illustrative. He moves unquestioningly between social norms and the like and individual conformity to them to a greater or lesser extent. We have both the social and the individual, the extensional and the intensional in set-theoretic terms. There is a presumption that these are consistent with one another. But consider the specific example of (the economics of) identity addressed by Akerlof and Kranton (2000). By appeal to the metaphor of choosing to be red or green, groups (ie sets of individuals) are formed, extensional. But identity is a property, greenness or redness, intensional. Whatever its other merits as a theory of identity (and it falls foul of what the meaning of identity is and how it is constructed and construed), the approach is caught out by Russell's paradox. Perhaps the easiest way to see this is in terms of the set N of those who do not have an identity. They do then, of course, have the identity of not having an identity! Be this as it may, individuals surely relate to a property of identity as well as constituting themselves in relation to it.¹¹

But we digress from the point of motivating the more formal idea that the institutional structure of our simple society potentially includes at least the following $\{a, \{a, b\}\}$, $\{b, \{a, b\}\}$ and $\{a, b, \{a, b\}\}$, together with our two individuals $\{a\}$ and $\{b\}$.¹² But we have no reason to stop at this point. The defining characteristic of an institution, or the social more generally, is that it relates to other institutions (and to individuals). Consequently, we can carry on building up institutions indefinitely. Linguists might study our current set of sets just listed, but they might form themselves into a professional institution, and they themselves might be studied by educationalists, governing and funding bodies, and so on. In principle, even on the basis of a two-person society, the potential for institutional structure grows indefinitely. An institution of the sets just listed can form a new institution, similarly with these,...

At what point do we call this process to a halt as surely we must in order to be practicable, although it should be observed that maximal institutions are liable to be the most powerful, not the most marginal in that they condition all others? For the sake of argument, let X be some sort of maximal institution (that might not be unique as in a system of nation-states). We might have the state in mind for example, or ideology, custom or culture. Now if X is to have any effect on society, it must interact with other institutions or individuals, with a say, just as individuals interact with the state, custom or culture. Otherwise, we do not have an institution as such, only its constituent members, totally disconnected from "society". In other words, (lower level) institutions and individuals have to interact with an institution; otherwise it is an institution in name alone. It follows that $\{X, a\}$ is an institution, contradicting the presumption that X is maximal.

In other words, it follows that the institutional structure is boundless. Is this some sort of trick? If I relate to the state, this relationship is a super-state to which I also relate and so on. The problem is that if we both base ourselves on methodological individualism and wish to construct an institutional structure that is distinct from the individuals themselves, that institutional structure does expand without limit. If it did have a limit, an individual could not relate to it without creating a contradiction. And, otherwise, if individuals do not relate to the highest level of institution, it is not clear how that institution could ever change or be replaced. Of course, we could fall back upon the infamous position of Mrs Thatcher, that there is no such thing as society just a collection of individuals (or families). But this would be against the spirit of the NIE which seeks to construct a notion of society distinct from, but built upon, its individuals. In other words, if we try and get, as it were, at the institution "from below", through the aggregate interactions of individuals, and by analogy with the iterative or extensional conception of set,¹³ then we are unable to obtain institutions with relational properties without potentially generating inconsistency.

Does this matter? Consider now a society made up of any number of individuals. As before, an institution is defined by the agents that make up its constituent membership, and who interact with one another, with individuals as starting point. Let I be a set of institutions a member of which is defined by its not being a constituent member of itself. Formally, $I = \{i: i \text{ belongs not to } i\}$. Does I belong to itself or not? If it does, then I is an institution that belongs to an institution (itself) and so it does not belong to I by definition. If, however, I does not belong to itself, then it does belong to itself.

Either way we have an inconsistency. In other words, by allowing all possible institutions to be built up out of constituent elements (or individuals), we create a contradiction in constructing an institution that neither interacts with itself nor does not interact with itself. By analogy with responses to Russell's paradoxes, the extensional and intensional definition of institutions are mutually inconsistent unless subject to restrictions. But it is not clear why we should prohibit certain types of institutions at the outset, even without knowing what they would be, just in order to avoid set-theoretic inconsistency. Nor would it appear to be reasonable, both in principle and in practice within the literature, to resolve this conundrum purely in favour of the extensional approach, thereby denuding the institutional of generic properties as reflected in the notions of custom, ideology, compulsion, culture, identity, and the like.

As a further, specific example, consider the case of money. This is often thought of as an institution from a variety of perspectives. It involves trust, financial organisations and governance, customary practice, and so on. Money raises two issues – what it is and how does it arise? It is also attached to a social property, as is recognised in the terminology of liquidity. From the perspective of methodological individualism, money arises out of accumulated acts of (potential) exchange. The liquidity of a particular good, m , say, is represented by the goods with which it can always exchange $\{g_1, g_2, \dots, g_n, \dots\}$. Typically, money in general is more broadly exchangeable against other goods and so attached to larger sets in this respect, with non-moneys only exchangeable with moneys. As is recognised in the definition of money in practice, some moneys are more liquid than others, and moneys may have different, if overlapping, spheres of exchangeability. Sometimes credit cards will serve where cash will not, and sometimes only cash will do. And, in principle and in practice, some moneys exchange not only against goods but against other moneys, so such moneys are exchangeable against goods and the set of goods that represents the liquidity of other moneys. Thus, some moneys will not only dominate others but also be able to purchase them and not necessarily vice-versa. In this respect, m_1 , say, can be considered to include m_2 , say, in its exchangeability set which is itself representative of an exchangeability set. Once again, the institutional structure of money is comprised of sets of sets and not just sets of goods.

Now liquidity, as mentioned, is a social property reflecting capacity to purchase and, by the same token, inability to purchase. So application of the notion of liquidity must allow for both liquidity and illiquidity. Accordingly, let M be the set of moneys (or goods if you prefer) that are not liquid with themselves. Does M belong to itself? If it does, it does not and vice-versa. We have an inconsistency as before. Expressed in terms of the social property of liquidity, if a money is liquid, it can buy anything including its own capacity to purchase. This will contradict any notion of money/liquidity built up out of its constituent or elemental acts of exchangeability, since a “maximal” money will be self-contradictory, both able and unable to purchase itself. This is not so fanciful as it seems. Is there a banknote that could purchase all other banknotes? We suspect not because it would have to be able to purchase itself and more, and so be more valuable than itself!

4 Broader Considerations

The results of the previous discussion are very simple and can be derived formally at least by virtue of a few lines of set theory alone. What comes in addition is offered more by way of background and motivation for what is after all, the loosest and most general definition of an institution (merely those who participate within it). Of course, the mathematical results, as opposed to their institutional interpretation, are not new. Significantly, as seen, the issues concerning the relative merits of individualistic or elemental and holistic or property approaches have themselves been debated within the mathematics and philosophy literature in ways that shed considerable light on the case for and against methodological individualism. Indeed, it is only necessary to make marginal changes to the language used in transposing arguments from the mathematical/philosophy/natural science domain to that of the social sciences.

In the specific case of institutions, or the social more generally, the iterative or individualistic approach is entirely inappropriate. It would mean that individuals have priority that is irreflexive and asymmetrical, precluding a full portfolio of feedbacks from institutions to those individuals. Or institutions are confined to simple types for which (more or less arbitrary) mixed types are excluded. Or institutions at a higher level cannot interact with those (and individuals) at a lower level. Surely intuition of what comprises an institution would rule out such artificial devices, not least because the cleverest of neoclassical economists would not have thought of them other than in order to rule out

contradictions. More generally, if economics confines itself to the iterative conception, equivalent to mathematics as pure science as opposed to its intuitive application to the real world, then it cannot proceed beyond the deductive tautologies or mathematical truths to appropriate knowledge or logic of the economy and its institutions consistently.

Thus, with Russell's paradoxes translated into institutional terms, we cannot without restriction both have institutions based on methodological individualism and institutions defined by their social properties. This does not mean that methodological individualism is refuted but it does imply that it cannot be extended to address institutions in a way that allows for social properties independent of those individuals. In this sense, Russell's paradoxes offer the conclusion of the general impossibility of the new institutional economics. Institutions cannot be derived that have social properties other than as a collection of individual interactions, unless certain institutional forms are precluded at the outset on the non-institutional grounds of pre-empting mathematical inconsistency. The alternative is to reject the element-based approach, or methodological individualism, and begin with properties, or social relations, independent of their individual elements, that is a systemic or holistic approach. This is, of course, anathema to mainstream economics and offers one explanation, or motivation, for why the implications of Russell's paradoxes for (the new institutional) economics should not have been explored despite the pride of place that mathematics and logic are purported to occupy within the mainstream.

5 Concluding Remarks

As misunderstandings have arisen in comments on earlier versions of this paper, it makes sense to emphasise what it does not seek to do. It does not seek to resolve Russell's paradoxes, to provide a contribution to the philosophical foundations of mathematics (or, indeed, to assert that these are either necessary or unavoidable).¹⁴ Nor does it establish a position on the relationship between mathematics and either economic theory or the economy. Nor do we disprove the possibility of methodological individualism and its application to the new institutional economics in pure or mixed forms. Rather, the intention is merely to point out the implications of certain arguments within set theory for the logical (in)consistency of deploying mixed notions of micro and macro, of the individual and the social.

Nor it should be added, am I claiming that the corresponding consequences are in some sense the most powerful form of criticism of the mainstream either in principle or in practice. They derive from axiomatics, other critiques do not, but have been overlooked. This does suggest, however, that what drives mainstream economics is not mathematical rigour or logic as such but these only in so far as they are consistent with an unquestioned set of techniques that are all too familiar – those involving optimising individuals, with given production and utility functions, and in search of equilibrium, for example, Moscati (2005).¹⁵ The technical apparatus is hedged with convexity assumptions and the like that the unsuspecting economist would not have thought of but for the need to grind out equilibrium with efficiency properties. This is the key to how the mainstream will react to this contribution or how it would respond to Russell's paradoxes if brought to their attention. They will simply be ignored, or observed only in passing on regardless often with accompanying misinterpretation, as for all contributions that do not conform to continuing use of standard techniques.¹⁶

This is not to say that the mainstream has been totally free of considerations that arise from the potential inconsistencies between extensional and intensional methods. These are exposed as extensive analysis from the individual exposes its own intensive limitations. It all began with the fiction of the Walrasian auctioneer – all price-takers means no price-makers. The problem of liquidity has already been discussed. The relationship between micro and macro remains a sore point, highlighted rather than resolved by the representative individual, and the same applies more generally to the various aggregation problems. Observance in the breach, if not ignorance, has been the fate, for example, of Arrow's Impossibility Theorem within social choice theory and, even more destructive for much of the intuition of the mainstream, the Cambridge Critique of Capital Theory. Nonetheless, to know that we cannot have a social welfare function based on pair-wise comparisons, a theory of production and distribution based on aggregate capital, and a theory of the social or institutional based on methodological individualism is of continuing critical relevance. Let these results be broadcast broadly, especially where the mainstream proclaims its superior rigour and scientific status. As with many other aspects, this is to work with notions of mathematics and science that have been rejected by mathematicians and scientists themselves for a hundred years and more.

Footnotes

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² See <http://www.soas.ac.uk/departments/departmentinfo.cfm?navid=490>

³ See Mosini (2007) for the huge difference between economics and the natural science, not least in the latter's inductive checks on the realism of its assumptions and its conceptualisation of equilibrium as a state of tension rather than as rest.

⁴ See collection edited by Dow (1998), for example, and Chick and Dow (2001).

⁵ We leave others to investigate the implications of other results within the logic of formal systems, such as Gödel's incompleteness theorems. There is also the issue of what sort of mathematics and mathematical assumptions are appropriate to economics – continuity, computability, and so on. See Velupillai (2005) for a contribution and other references of interest.

⁶ Giocoli (2005) for a partial exception.

⁷ In their Preface, they note that, "we have tried also to narrow the range of *philosophical* issues discussed in the selection to ones that could most easily be recognized as concerning the philosophy of *mathematics*", p. vii, so there is little scope for explicit consideration of extension to the social sciences.

⁸ Even more simply, for Wang (1983, p. 530), if we substitute institution for set:

A set is a collection of previously given objects; the set is determined when it is determined for every given object x whether or not x belongs to it. The objects which belong to the set are its members, and the set is a single object formed by collecting the members together.

⁹ As is recognised in the formal theory of networks, themselves to be considered a form of institution, with networking across networks, etc.

¹⁰ See Fine (1980) and Hodgson (2007) for the failure of methodological individualism in practice for always having to assume as prior something as social for individuals to work upon.

¹¹ For a more extended critique of Akerlof and Kranton (2000) on this and other issues, see Fine (2007a).

¹² It is a moot point whether individuals $\{a\}$ and $\{b\}$ are themselves institutions or not. Simple-minded ruling out of inner speculation would suggest that a and $\{a\}$ are the same as one another. But allowing reflection from society back upon oneself would forge a difference between the two as an external influence, albeit one itself forged presumably out of aggregated individual interaction. The Robinson Crusoe metaphor for decentralised general equilibrium, for example, allows for the institution of the market independent of Robinson himself but with which he interacts as both producer and consumer. No doubt, he also talks to himself. Note that the fanatically consistent purveyor of the economic approach to all social science, Gary Becker (1996, p. 18), is mathematically correct to deny individual's the right to meta-preferences but is formally nonsensical in attaching a lack of reason to rationality, Sen (1977)!

¹³ To pursue the analogy, Putnam (1983, p. 310) concludes:

The real significance of the Russell paradox ... is this: it shows that *no* concrete structure can be a standard model for the naive conception of the totality of all sets; for any concrete structure has a possible extension that contains more "sets".

¹⁴ See Putnam (1983).

¹⁵ And hence a shifting meaning in what is meant by mathematical argument and rigour to conform to the dictates of mainstream economics, Weintraub (1998).

¹⁶ Note the striking parallel with the response of reformists and apologists to Russell's paradoxes (heterodoxy and orthodoxy, respectively, as far as economics is concerned), Benacerraf and Putnam (1983, p. 3):

One way of describing the differences between these two groups is to say that, for one group, the epistemological principles have a higher priority or centrality than most particular bits of mathematics, and hence can be used as a critical tool; whereas for the other group just the reverse is the case: Existing mathematics is used as a touchstone for the formation of an epistemology, one of whose conditions of adequacy will be its ability to put all of mathematics in the proper perspective. To put it somewhat crudely, if some piece of

mathematics doesn't fit the scheme, then a writer in the first group will tend to throw out the mathematics, whereas one in the second will tend to throw out the scheme.