Debt cycles, instability and fiscal rules: a Godley-Minsky synthesis

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Abstract: Wynne Godley and Hyman Minsky were two macroeconomists who 'saw the crisis coming'. This paper develops a simple macrodynamic model that synthesises some key perspectives of their analytical frameworks. The model incorporates Godley's financial balances approach and postulates that private sector's propensity to spend is driven by a stock-flow norm (the target net private debt-to-income ratio) that changes endogenously via a Minsky mechanism. It also includes two fiscal rules: a Maastricht-type fiscal rule, according to which the fiscal authorities adjust the government expenditures based on a target net government debt ratio; and a Godley-Minsky fiscal rule, which links government expenditures with private indebtedness following a counter-cyclical logic. The analysis shows that (i) the interaction between the propensity to spend and net private indebtedness can generate cycles and instability; (ii) instability is more likely when the propensity to spend responds strongly to deviations from the stock-flow norm and when the expectations that determine the stock-flow norm are highly sensitive to the economic cycle; (iii) the Maastricht-type fiscal rule is destabilising while the Godley-Minsky fiscal rule is stabilising; (iv) the paradox of debt can apply both to the private sector and the government sector.

Keywords: Godley; Minsky; debt cycles; instability; fiscal rules

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1. Introduction

Wynne Godley and Hyman Minsky were two macroeconomists who 'saw the crisis coming'.[1] In 1999 Godley published his well-known article on the 'seven unsustainable processes' in the US economy (Godley, 1999). In this article he argued that the rising indebtedness of the US private sector was unsustainable and, therefore, private expenditures could not be considered as a source of steady growth in the medium run. He also pointed to the unsustainability of the rising US net foreign indebtedness. Using a stock-flow consistent analytical framework, Godley argued that without a change in fiscal policy stance or an important rise in net exports, the US economy was doomed to witness a severe recession and a sharp rise in unemployment. These warnings were repeated in his publications as a head of the Levy Economics Institute's macro-modelling team (see, e.g. Godley, 2003, 2005; Godley *et al.*, 2005). The 2007-9 crisis verified Godley's fears: the US economy contracted sharply and the unemployment rate increased substantially.[2]

Minsky (1975, 1982, 2008) developed a theory that explains how indebtedness can increase in periods of tranquillity as a result of endogenous forces that reduce the desired margins of safety of economic units. This gradual reduction in the desired margins of safety was considered by Minsky as the reason behind the increasing financial fragility that accompanies economic expansion and periods of stability. According to his financial instability hypothesis, the increasing fragility makes the macro systems more prone to shocks that reduce the ability of borrowers to repay their debt. These shocks can lead to severe economic recessions. The processes described in Minsky's analysis are broadly in line with the

pre-crisis developments in the US and other advanced economies that ultimately led to the Great Recession.

The emphasis that Godley and Minsky placed on financial relationships as sources of cycles and instability enabled them to provide some very important insights into the dynamics of modern macroeconomies. However, they did so from quite different angles. Godley concentrated more on the macroeconomic relationships between the private, the government and the foreign sector and postulated that in the medium to long run the fluctuations in financial balances and growth are driven by some exogenous stock-flow norms. Minsky, on the other hand, focused more on the relationships within the private sector (primarily on the financial relationships between firms and banks) and explained the macroeconomic fluctuations by considering endogenous changes in norms and valuations of risk.

Although it is widely held that Godley's and Minsky's perspectives are both important for the explanation of macroeconomic dynamics, there is still a lack of a formal framework that synthesises them. On the one hand, there is a large literature on formal Minskyan models that capture various dynamics related to Minsky's financial instability hypothesis; see, for example, Taylor and O'Connell (1985), Chiarella and Di Guilmi (2011), Passarella (2012), Keen (2013), Ryoo (2013A), Nikolaidi (2014), Charles (2016) and the references therein. However, none of these models include explicitly insights from Godley's projection analyses. On the other hand, there are some theoretical stock-flow consistent models that utilise aspects of Godley's projection approach, but make no explicit links to Minskyan dynamics (e.g. Godley and Lavoie, 2007; Martin, 2008; Leite, 2015).

The purpose of this paper is to develop a simple macrodynamic model that makes a Godley-Minsky synthesis. The model concentrates on certain aspects of Godley's and Minsky's approaches that are deemed more important for a simplified explanation of debt cycles and instability in a national

macroeconomy. The key features of the model and the principal results of the analysis are the following.

First, as in Godley's projection analyses, the model economy consists of three sectors: the private sector, the government sector and the foreign sector. This permits an explicit consideration of Godley's financial balances approach that explains the interlinkages between these sectors and the resulting effects on debt accumulation and growth. Remarkably, Godley's financial balances approach is broadly in line with Kalecki's profit equation that was used by Minsky.

Second, drawing on Godley, the private sector's propensity to spend is driven by a stock-flow norm (the target net debt-to-income ratio). It is shown that, under certain conditions, the interaction between the propensity to spend and net private indebtedness generates cycles and instability. Instability is more likely when the propensity to spend responds strongly to deviations from the stock-flow norm. A paradox of debt result arises: the more the private sector and its lenders attempt to put net private indebtedness under control, by adjusting private expenditures, the more the net private debt ratio destabilises.

Third, following Minsky, it is assumed that the stock-flow norm varies endogenously as a result of changes in the expectations and the conventions of borrowers and lenders during the economic cycle. It is shown that this endogeneity can give rise to Godley-Minsky debt cycles and it is a source of instability. Instability is more likely when the stock-flow norm is highly sensitive to the economic cycle.

Fourth, two different fiscal rules are introduced: a Maastricht-type fiscal rule, according to which the fiscal authorities adjust the government expenditures based on a target net government debt ratio; and a Godley-Minsky fiscal rule, which links government expenditures with private indebtedness following a counter-cyclical logic. Simulation analysis illustrates that the Maastricht-type fiscal rule is destabilising

while the Godley-Minsky fiscal rule is stabilising. Apart from supporting the view that counter-cyclical fiscal policy has positive effects on the stabilisation of output, this result suggests that counter-cyclical fiscal policy is also conducive to the stabilisation of government indebtedness. Moreover, it is shown that the paradox of debt can apply to the government sector: the more the fiscal authorities attempt to target a specific government debt ratio, by adjusting the government expenditures, the more this ratio destabilises.

The paper proceeds as follows. Section 2 lays out the structure of the model. Section 3 explores the interaction between private sector's propensity to spend and net private indebtedness when the target net private debt-to-income ratio and government expenditures are exogenous. Section 4 endogenises the target net private debt-to-income ratio and studies how Godley-Minsky cycles can arise. It also examines the implications of this endogeneity for instability. Section 5 introduces fiscal rules and analyses their (de)stabilising effects. Section 6 summarises and concludes.

2. Structure of the model

We consider an economy that consists of three sectors: the private sector (which includes households, non-financial firms and banks), the government sector and the foreign sector. The transactions that take place between the sectors of the economy are shown in Table 1. The balance sheet matrix, which depicts financial assets/liabilities, is shown in Table 2. The symbols used in the model are reported in Appendix A. The identities of the model that stem from the transactions/balance sheet matrices and national accounting are presented in Appendix B1.

<Insert Table 1 here>

<Insert Table 2 here>

The total output of the economy is determined by aggregate expenditures (see equation B1 in Appendix B1). All sectors of the economy accumulate financial assets and financial liabilities. The net debt of each sector is equal to its financial liabilities minus its financial assets. A sector is a net debtor when its net debt is positive and a net creditor when its net debt is negative.

The private sector invests in tangible capital goods. If the saving of this sector is not enough to cover the investment expenditures, net private debt increases (equations B3 and B4 in Appendix B1). The government sector collects taxes and makes primary expenditures by purchasing goods produced by the private sector. When total expenditures (which include net interest payments) are higher than taxes, net government debt increases (see equation B5 in Appendix B1). The private sector has trade and financial relationships with the foreign sector. The net debt of the foreign sector increases when the trade balance (exports minus imports) is higher than the net interest income received from the foreigners (see equations B6 and B7 in Appendix B1).

For simplicity, the following assumptions have been made: the interest rate is exogenously determined by the monetary authorities and is the same for both the private and the government net debt;[3] the price level is set equal to unity; there are no changes in asset prices and exchange rates that could affect the value of assets and liabilities (and, thus, the value of net debt).

Due to national accounting, the sum of the net debt of the three sectors of the economy is equal to zero (see Table 2 and equation B7 in Appendix B1). This, for example, means that when the net debt of the domestic economy declines, the net debt of the foreign sector increases. National accounting also implies that the sum of the balances of the three sectors of the economy is equal to zero (see equation B8 in Appendix B1).[4] This reflects Godley's financial balances approach and implies that the balance of one sector cannot improve without a deterioration in the balance of at least one of the other two

sectors.[5] For example, if the government sector desires to decrease its deficit to a specific level then the private sector and/or the foreign sector should be willing to accept a deterioration of their balances in an accurately offsetting manner. Otherwise, the intended decline in deficit cannot be attained. Moreover, since most components of the financial balances are also components of the aggregate demand, any attempt of the sectors to improve their balances may lead to lower output if the other sectors do not desire to experience lower balances. The financial balances approach has been widely used by Godley himself and other scholars to analyse the macroeconomic developments in the US and other economies (see e.g. Godley *et al.*, 2007; Zezza, 2009; Brecht *et al.*, 2012; Sawyer, 2012; Wolf, 2012).

A distinguishing feature of the financial balances approach is the consolidation of households, firms and banks into one single private sector. This implies that in our model the transactions between households, firms and banks are not taken explicitly into consideration. Moreover, the assets and the liabilities of the private subsectors that are counterparts of the assets and liabilities of other private subsectors are netted out in the estimation of net private debt. The net private debt refers, therefore, solely to the net liabilities of the private sector that are net assets of the government and the foreign sector. Although this consolidation is a great simplification with various limitations (see Dos Santos and Macedo e Silva, 2010 and Martin, 2012 for a discussion), it has proved quite useful in Godley's projections and other empirical analyses that focus on the interaction between private sector's behaviour, fiscal policy and foreign balance. Moreover, it serves the purposes of our simple skeleton that intends to capture the dynamics of a national macroeconomy by using a high-level aggregation.

In the model private expenditures (P) include the consumption and the investment expenditures of households and firms. Private sector's propensity spend out of its income (p) is defined as the ratio of private expenditures to private sector's income:[6]

$$p = \frac{P}{Y - T} \tag{1}$$

where Y is the output of the economy and T denotes taxes.

Equation (2) defines the primary government expenditures (G)-to-output ratio:

$$g = \frac{G}{Y} \tag{2}$$

The net private debt (D_P)-to-income ratio is given by:

$$d_P = \frac{D_P}{Y - T} \tag{3}$$

Equation (4) gives the net government debt (D_G)-to-output ratio:

$$d_G = \frac{D_G}{Y} \tag{4}$$

As in most Godley's models, taxes and imports (M) are proportional to the output of the economy ($\tau, m > 0$):

$$T = \tau Y \tag{5}$$

$$M = mY \tag{6}$$

Moreover, exports (X) are assumed to grow at an exogenously given rate, g_X (see, e.g., Godley and Lavoie, 2007, p. 98):

$$\dot{X} = g_X X \tag{7}$$

This exogenous rate relies on factors such as the economy's structural competiveness and the income of the foreign sector, which are taken as given.

The behaviour of the private sector is determined using insights from both Godley and Minsky. Godley argued that the private sector targets in the long run a specific stock of net financial assets as a proportion of its disposable income (a stock-flow norm). He also postulated a formula which states that the balance of the private sector adjusts in order for this desired stock to be attained (see Godley and Cripps, 1983; Godley, 1999; Godley and Lavoie, 2007).[7] In our model this idea is captured by the following equation:

$$\dot{p} = \lambda[p^B + \Xi\left(d_P^T - d_P\right) - p] \tag{8}$$

where p^B is a benchmark value of the propensity to spend which is used as a reference, d_P^T is the target net private debt-to-income ratio, Ξ is a positive function of $(d_P^T - d_P)$ – hence $\Xi_{d_P^T} > 0$ and $\Xi_{d_P} < 0$ – and λ is an adjustment parameter ($0 < \lambda < 1$). The private sector has a desired propensity to spend, captured by the term $p^B + \Xi\left(d_P^T - d_P\right)$. $\Xi\left(d_P^T - d_P\right) = 0$ when $d_P^T = d_P$, $\Xi\left(d_P^T - d_P\right) > 0$ when $d_P^T > d_P$ and $\Xi\left(d_P^T - d_P\right) < 0$ when $d_P^T < d_P$. Hence, when d_P^T is higher (lower) than d_P, the desired propensity to spend is higher (lower) than the benchmark value, p^B, since the private sector desires to spend more (less) in order to achieve the target debt ratio. According to equation (8), there is a gradual adjustment of the actual propensity to spend towards the desired one.

Three points are in order. First, we assume that the function $\Xi\left(d_P^T - d_P\right)$ is continuous, differentiable and bounded from above and below. The boundedness is justified by the fact that the propensity to spend of the private sector cannot deviate significantly from the benchmark value that is used as a reference. Hence, as the difference between d_P^T and d_P increases, the impact of the target debt ratio on the propensity to spend diminishes because the private sector wishes to keep the propensity to spend relatively close to its benchmark value.

Second, d_P^T is not only set by the private sector itself. It is also set by the government and, most importantly, by the foreign sector that are potentially lenders/borrowers of the private sector. For instance, a rise in d_P^T might capture a rise in the willingness of foreign investors to lend to the private sector of a national economy (households, firms or banks). Therefore, a change in the propensity to spend reflects the decisions of both the private sector and its lenders/borrowers. Interestingly, equation (8) can capture changes in private expenditures caused by capital inflows and capital outflows. For example, the inequality $d_P^T > d_P$ may reflect periods in which the net debt of the private sector is considered by foreign lenders as sufficiently small. In such periods the existence of a low perceived lender's risk induces higher capital inflows that lead to higher private expenditures relative to income. On the other hand, the inequality $d_P^T < d_P$ may capture periods of capital outflows in which the lender's risk is perceived to be high. [8]

Third, although a change in the propensity to spend is the primary means through which the private sector can affect its net indebtedness, it may not have the desired outcomes. As will become clear below, the decision for spending affects the output of the economy and, therefore, has feedback effects on the net private debt-to-income ratio.

Godley postulated that the stock-flow norm is exogenous. However, such an assumption can be considered satisfactory only as a first approximation. In reality, economic units' desired margins of safety, which determine the stock-flow norm, change endogenously during the economic cycle. Minsky (2008, pp. 193, 209) argued that during periods of expansion – when the outstanding debts are serviced without significant problems – the desired margins of safety of borrowers and lenders decline. This happens because the recent good performance of the economy and the favourable credit history induce economic units to accept financial structures that were previously assessed as risky. The opposite holds in periods in which the economic performance and credit history are not favourable.[9]

Although Minsky's arguments primarily refer to the behaviour of firms and banks, they can apply to any borrower-lender relationship and, therefore, to the financial relationships between the private sector of a national economy and its lenders/borrowers (the government and the foreign sector). Hence, drawing on the above-mentioned perspectives, we postulate the following specification:

$$\dot{d}_P^T = \theta[d_P^B + Z(g_Y - g_{Y0}) - d_P^T] \tag{9}$$

where g_Y is the actual economic growth, g_{Y0} is the normal growth rate, Z is a positive function of $(g_Y - g_{Y0})$ – hence $Z_{g_Y} > 0$ and $Z_{g_Y} < 0$ –, d_P^B is a benchmark value of the debt ratio which is used as a reference and θ is an adjustment parameter ($0 < \theta < 1$). The term $d_P^B + Z(g_Y - g_{Y0})$ captures the debt ratio which is deemed as an acceptable target based on the contemporary economic, social and institutional conditions. We have $Z(g_Y - g_{Y0}) = 0$ when $g_{Y0} = g_Y$, $Z(g_Y - g_{Y0}) > 0$ when $g_Y > g_{Y0}$ and $Z(g_Y - g_{Y0}) < 0$ when $g_Y < g_{Y0}$. In other words, when economic growth is higher (lower) than the growth rate conceived as normal, the acceptable target debt-to-income ratio of the private sector is higher (lower) than the benchmark debt ratio, d_P^B.[10] Equation (9) shows that, due to conventional reasons, the target debt ratio gravitates slowly towards the acceptable target debt ratio.

The positive impact of economic growth on the target debt ratio introduces a self-reinforcing mechanism similar to the one suggested by Taylor (2004, p. 303). In his formulation, Taylor assumes that due to confidence reasons autonomous investment increases more when the investment rate is already high. Similarly, in our model, economic growth is driven up when economic growth is already high since favourable expectations lead borrowers and lenders to target a higher debt ratio, making private spending higher. As pointed out by Lavoie (2014, p. 446), Taylor's (2004, p. 303) formulation draws on the model of Taylor and O'Connell (1985) – the first attempt to formalise Minskyan ideas. In their model, Taylor and O'Connell (1985) introduced a 'state of confidence' variable and showed how the interaction of expectations with asset prices can produce Minskyan dynamics. Although the crucial role of asset prices is not incorporated in our model,[11] the Minskyan idea that expectations can produce destabilsing forces is a common feature between our model and the model of Taylor and O'Connell (1985).

In equation (9) the function $Z(g_Y - g_{Y0})$ is assumed to be continuous, differentiable and bounded from above and below. The boundedness implies that the acceptable target debt ratio cannot deviate substantially from the benchmark net debt ratio which is used as a reference. Hence, when economic growth becomes very high or very low compared to the normal growth rate, the changes in the acceptable target debt ratio become small.

The benchmark net debt ratio depends on deep economic, institutional and social factors (e.g. the degree of financialisation, the degree of openness of the domestic financial system, the prevailing consumption and investment norms, the society's perception of the role of debt, etc.). When $d_P^B > 0$, the private sector has a net debtor benchmark position; when $d_P^B < 0$, it has a net creditor benchmark position. The benchmark net debt ratio is expected to change over a long-run horizon. For example, over the 1990s and the 2000s many advanced economies experienced a sustained reduction in the

balance of their private sector (see, for example, Tymoigne and Wray, 2014, pp. 107-112). It is very likely that this reduction partially reflected deep economic, institutional and social economic changes that took place over this period, such as financial deregulation, financial globalisation and the gradual prevalence of norms that were conducive to higher levels of consumption and debt. These fundamental changes probably moved the private sector in these countries towards a net debtor benchmark position.

Overall, according to equation (9), the acceptable target net debt-to-income ratio depends on (a) cyclical changes in economic growth, reflected in $Z(g_Y - g_{Y0})$, and (b) deep long-run economic, institutional and social factors, captured by d_P^B. Following the distinction that Palley (2011) has made between the Minsky basic cycle and the Minsky super-cycle, it could be argued that the cyclical changes effects are related to the Minsky basic cycle, while the deep long-run factors have to do with the Minsky super-cycle which has a longer time horizon. In this paper, d_P^B is deemed constant and, therefore, emphasis is placed on the dynamics linked with the basic Minsky cycle. However, future extension of this model could allow d_P^B to change endogenously permitting thereby the combination of the Minsky basic cycle with the Minsky super-cycle.[12]

The government expenditures are determined by fiscal rules. Fiscal rules have been widely adopted over the past two decades or so. These rules impose constraints on fiscal aggregates, such as government debt and deficit (see IMF, 2009; Schaechter *et al.*, 2012). We first consider a simple Maastricht-type rule which states that primary government expenditures (relative to output) decline when the net government debt-to-output ratio is higher than a specific target (d_G^T). Formally, this rule is written as follows:

$$\dot{g} = \mu\left(d_G^T - d_G\right) \tag{10A}$$

where $\mu > 0$. Note that the essence of this rule would not change if the tax rate was used as a tool instead of the government expenditures.

We then consider an alternative fiscal rule that departs from the conventional approach since it places no limits on any specific fiscal aggregate. On the contrary, its rationale is that fiscal policy should stabilise the macroeconomy by increasing (decreasing) government expenditures when the private sector exerts contractionary (expansionary) pressures as a result of its attempts to reduce (increase) its indebtedness. Formally:

$$\dot{g} = -\kappa_1\left(d_P^T - d_P\right) + \kappa_2\left(g_0 - g\right) \tag{10B}$$

where $\kappa_1, \kappa_2 > 0$. The second term has been introduced to reflect the fact that government expenditures cannot deviate significantly from a benchmark value, g_0. Equation (10B) is consistent with the perceptions of both Godley and Minsky who emphasised that the government should intervene to offset fluctuations in economic activity that stem from the inherently unstable behaviour of the private sector.[13] We thus call equation (10B) the 'Godley-Minsky fiscal rule'. It should be pointed out that this rule is generally in line with the Keynesian counter-cyclical fiscal policy.

As shown in Appendix B3, the growth rate of the economy is given by:

$$\frac{\dot{Y}}{Y} = g_Y = g_X + \frac{\dot{p}(1-\tau) + \dot{g}}{1 + m - p(1-\tau) - g} \tag{11}$$

Using equations (B2), (B4), (1), (3), (5) and (11), we get:

$$\dot{d}_P = \frac{\dot{D}_P}{Y(1-\tau)} - d_P\frac{\dot{Y}}{Y} = p - 1 + \left(r - g_Y\right)d_P \tag{12}$$

Employing equations (B5), (2), (4), (5) and (11), we get:

$$\dot{d}_G = \frac{\dot{D}_G}{Y} - d_G \frac{\dot{Y}}{Y} = g - \tau + (r - g_Y) d_G \tag{13}$$

Equations (12) and (13), in conjunction with (11), show that, when the private and the government sector decide to spend less (relative to income), the impact on their net debt-to-income ratios depends on whether they are net debtors or net creditors. If the net debt is positive, there are two counteracting effects. On the one hand, the decline in expenditures (i.e. in p and g) tends to reduce the net debt-to-income ratios. We call this the 'spending effect'. On the other hand, such a decline reduces g_Y; this places upward pressures on the positive net debt-to-income ratios by decreasing the denominator of these ratios. We call this the 'growth effect'. However, when the net debt is negative, these two effects are mutually reinforcing. The reason is that a lower g_Y reduces the denominator in the negative net debt-to-income ratios making them more negative.

Overall, the model can be summarised as a 5D (4D) dynamic system, which is reproduced below for convenience:[14]

$$\dot{d}_P = p - 1 + (r - g_Y) d_P \tag{12}$$
$$\dot{p} = \lambda [p^B + \Xi (d_P^T - d_P) - p] \tag{8}$$
$$\dot{d}_P^T = \theta [d_P^B + Z (g_Y - g_{Y0}) - d_P^T] \tag{9}$$
$$\dot{d}_G = g - \tau + (r - g_Y) d_G \tag{13}$$
$$\dot{g} = \mu (d_G^T - d_G) \tag{10A}$$
$$\dot{g} = -\kappa_1 (d_P^T - d_P) + \kappa_2 (g_0 - g) \tag{10B}$$

The unique steady state of the system is reported in Appendix B4. In the next sections the system will be analysed in three steps. In the first step (Section 3), the dynamic interaction between private sector's propensity to spend and net private indebtedness will be examined assuming that the target net private debt-to-income ratio and the government expenditures-to-output ratio are exogenous. In the second

step (Section 4), the target net private debt-to-income ratio will be endogenised. In the third step (Section 5), the fiscal policy rules will be introduced. Our examination relies on analytical solutions and numerical simulations. The parameter values used in the simulation exercises and their justification are reported in Appendix C.[15]

Prior to proceeding to this analysis, it is useful to point out that when economic growth is exogenous, the net private debt-to-income ratio and the net government debt-to-output ratio are stable when the growth rate of exports is higher than the interest rate (i.e. $g_X > r$; see Appendix B5). This implies that when economic activity is exogenous, the stability of the net debt ratios relies on the export performance of the economy and the stance of monetary policy. The lower the interest rate set by monetary authorities and the higher the export growth of the national macroeconomy, the higher the likelihood that the debt ratios will stabilise. In the dynamic analysis and the simulations that follow it will be assumed that $g_X > r$. This will allow us to focus on the destabilising forces that stem from the Godleyan and Minskyan mechanisms described above.

3. Interaction between private sector's propensity to spend and net private indebtedness

This section analyses the 2D subsystem consisting of equations (12) and (8). Our aim is to examine the dynamic interaction between private sector's propensity to spend and net private indebtedness when the target net private debt-to-income ratio and government expenditures are exogenous. As shown in Appendix D, when the private sector has a net creditor benchmark position, equation (8) does not produce destabilising forces: the stabilising impact of a change in the propensity to spend is reinforced by the associated 'growth effect' (see also Section 2).[16] On the contrary, when the private sector has a net debtor benchmark position, the likelihood that the system is unstable is higher the higher is the responsiveness of the propensity to spend to the divergence between the actual and the target net

private indebtedness (for a given $\Xi\left(d_P^T - d_P\right)$ this responsiveness is captured by λ). The rationale behind this result is straightforward: when the net private debt ratio is higher (lower) than the target one, any attempt of the private sector to reduce (increase) its indebtedness by reducing (raising) the propensity to spend has an adverse (favourable) impact on economic growth. When λ is higher than a specific threshold λ^*, this 'growth effect' dominates the 'spending effect' leading to instability. It is interesting to note that, as shown in Appendix D, λ^* increases when the interest rate declines or the growth rate of exports increases. This implies that adequate monetary and trade policy can, until some limit, prevent the destabilising forces that stem from the behaviour of the private sector.

<Insert Table 3 here>

The stability properties of the 2D system are summarised in Table 3. Figure 1a illustrates the stability that arises when $\lambda < \lambda^*$ and $d_P^B > 0$. When $\lambda \geq \lambda^*$, the system is unstable (see Figure 1b). However, since, as shown in Appendix D, the system is bounded, instability gives rise to limit cycles.[17] Figure 2 presents how these cycles are reflected in the interaction between private indebtedness and private sector's propensity to spend (we use the same parameter values as in Figure 1b). Suppose that the economy is initially in phase I. Since $d_P < d_P^T = d_P^B = 0.2$, the private sector increases its propensity to spend, producing higher than steady-state growth. Simultaneously, net private indebtedness declines because the propensity to spend is not high enough. Phase I can be interpreted as a phase of recovery. As the net private debt-to-income ratio declines, p continues to increase and eventually the economy enters phase II in which the propensity to spend is high enough to generate a rise in indebtedness. In this phase the economy continues to exhibit a high growth which, however, is accompanied by higher fragility. At some point, d_P becomes very close to d_P^T. At that point the indebtedness of the private sector is conceived to be extremely high from the borrowers' and/or lenders' perspective. This causes a reduction in private sector's propensity to spend. The economy enters a period of stagnation (phase III) where low growth coexists with rising net indebtedness. This rising indebtedness reduces further the

private sector's propensity to spend. Indebtedness starts declining only when the propensity to spend is low enough to outweigh the adverse affects of low growth on the debt ratio. When this happens, the economy enters a new phase (phase IV) where economic growth remains low (since d_P is still higher than d_P^T). However, declining indebtedness sets the stage for the recovery that occurs when d_P falls short of d_P^T. When this happens, a new cycle begins.

<Insert Figure 1a here>

<Insert Figure 1b here>

<Insert Figure 2 here>

The cycles depicted in Figure 1b and Figure 2 could be characterised as 'Godley cycles', since they stem from private sector's attempt to achieve an exogenous stock-flow norm by modifying its propensity to spend. These cycles are broadly in line with the empirical evidence provided by Koo (2013) according to which the saving behaviour of the private sector (motivated by the willingness to leverage or deleverage) was the principal driver of expansions and contractions in many advanced economies over the last decades.

The fact that a higher value of λ can lead to cycles and instability has an interesting interpretation. It implies that the more the private sector and its lenders attempt to put net private indebtedness under control by adjusting private expenditures, the more the private debt ratio destabilises. Arguably, this is a paradox of debt result. Although the adjustment of the propensity to spend seems to be a sensible behaviour for the control of private indebtedness, the resulting macroeconomic effects prevent the realisation of the desired indebtedness.[18]

Since g is constant in the 2D subsystem, net government indebtedness is exclusively driven by the 'growth effect': when economic growth is high (low) enough the net government debt-to-output ratio

declines (increases). As shown in Figure 2, this 'growth effect', which in the 2D subsystem is exclusively linked to the behaviour of the private sector, is sufficient to destabilise the net government debt-to-output ratio. Therefore, private sector's attempt to achieve its desired stock-flow norm can destabilise public indebtedness despite the fact the government sector is passive. However, the constancy of g has as a result that the government balance does not change significantly relative to the other two balances. Consequently, any deterioration or improvement in the balance of the private sector is almost entirely mirrored in the balance of the foreign sector.

4. Endogenising the targeted net private indebtedness

We now allow the target net private debt-to-income ratio to change endogenously according to equation (9); g is still kept at its steady-state value. As shown in Appendix E, the introduction of an endogenous change in the target net private debt-to-income ratio can transform an otherwise stable system of d_P and p into an unstable one. Instability arises when the responsiveness of the target net private debt-to-income ratio to changes in economic growth (which is captured by θ for a given $Z(g_Y - g_{Y0})$) is higher than a specific threshold θ^* (see Table 3). Figure 3 illustrates this in our simulations.[19] The figure refers to a private sector that has a net creditor benchmark position (similar results arise when a net debtor benchmark position is considered). It can be seen that the stability properties of the 3D subsystem change as θ increases: although the system is stable for low values of θ, it becomes unstable when θ becomes higher than $\theta^* \approx 0.05$.

<Insert Figure 3 here>

The underlying mechanism can be explained as follows. In periods of low growth, when net private indebtedness is high, the deterioration in borrowers' and lenders' expectations induces them to target a

lower net debt ratio than the benchmark one (d_P^B); recall that in the 2D subsystem $d_P^T = d_P^B$. Therefore, the difference between the actual and the target ratio increases, producing a greater decline in the propensity to spend (and therefore in economic growth) compared to the 2D subsystem. Inversely, in periods of high growth, in which net private indebtedness is low, the favourable expectations due to the good performance of the economy make the perceived risk lower. This leads to a higher target net debt ratio than the benchmark one and, hence, to a more significant rise in the propensity to spend. This results in higher economic growth. The greater fluctuations in both the propensity to spend and economic growth are reflected in the law of motion of the net debt ratio. If θ is sufficiently high, these fluctuations ultimately lead to instability.

As shown in Figure 4 and Appendix E, instability gives rise to perpetual cycles. The emergence of cycles stems from the fact that the desired propensity to spend and the acceptable target debt ratio cannot deviate substantially from their benchmark values. Hence, we overall have that an otherwise stable Godley system can exhibit cyclical behaviour when it is combined with the Minskyan endogeneity of the stock-flow norm.

<Insert Figure 4 here>

5. Introducing fiscal rules

We now turn to examine how the stability of the macro system changes when fiscal rules are introduced. Figure 5 illustrates the dynamic adjustment of the system when fiscal authorities adopt a Maastricht-type fiscal rule (equation 10A). Figure 6 shows the dynamic adjustments when a Godley-Minsky fiscal rule is implemented (equation 10B). In both simulation exercises the parameter values that refer to equations (8), (9) and (12) are the same with those used in the simulations presented in Figure 3. Moreover, the same range of values for θ has been employed and the case in which $d_P^B < 0$

has again been considered. This allows us to specify how the dynamic adjustments of the macro system are modified as a result of the introduction of fiscal rules.

Consider first the Maastricht-type fiscal rule. Comparing the simulation results between Figure 3 and Figure 5, it can be observed that instability increases as a result of the implementation of this fiscal rule: first, for high values of θ the cycles become much more intense; second, instability gradually arises even for low values of θ. Intuitively, the following mechanisms are at play. Whenever economic growth is low (as a result of high net private indebtedness) there is a tendency for the net government debt-to-output ratio to increase. At some point during the period of low growth, the government debt ratio becomes higher than d_G^T. To guarantee fiscal discipline the government responds by reducing the expenditures-to-output ratio. This magnifies the contractionary effects that stem from the behaviour of the private sector: at a first place, economic growth is adversely affected by the decrease in g; at a second place, this additional decline in growth enhances the deterioration in the expectations reducing further d_P^T; other things equal, the divergence between d_P and d_P^T increases with destabilising effects on growth, private expenditures and net private debt. When the private sector has a net debtor benchmark position there is an additional channel through which the difference between d_P and d_P^T increases: lower growth resulted from fiscal stance places upward pressures on d_P. The inverse mechanisms are at work when economic growth is high. This implies that the Maastricht-type fiscal rule increases the amplitude of debt and economic cycles.[20]

Importantly, the induced instability refers not only to the private sector but also to the government sector. Figure 5 illustrates that, as time passes, the Maastricht-type fiscal rule generates significant fluctuations in both the net government debt-to-output ratio and the government balance (as a proportion of output). These fluctuations are much more severe than those observed in Figure 3. This result stems from the amplification of the economic cycles described above. Therefore, in an economy in which the private expenditures respond to changes in net private indebtedness and the targeted

indebtedness is endogenous, the currently fashionable debt brake rules do not only seem to destabilise the private sector but they may also be ineffective in ensuring fiscal prudence. Actually, a paradox of debt result arises: the more the fiscal authorities attempt to target a specific government debt ratio, by adjusting the government expenditures, the more this ratio destabilises.[21]

<Insert Figure 5 here>

On the other hand, the Godley-Minsky fiscal rule suggested here is capable of stabilising both the private economy and the government sector for high values of θ, playing a similar role as the traditional Keynesian counter-cyclical fiscal policy. Figure 6 indicates this. After some fluctuations in the initial periods (which are much less intense than the fluctuations in Figure 5) all macro variables converge towards their steady-state values. Economically, this can be explained as follows. When economic growth is low due to high net private indebtedness, the implementation of the Godley-Minsky fiscal rule produces a rise in the government expenditures-to-output ratio. This has favourable effects on economic growth since it tends to reduce the divergence between d_P and d_P^T by placing upward pressures on d_P^T. As alluded to above, the reduction of this divergence is conducive to stability. In high-growth phases the government expenditures-to-output ratio falls, slowing down the economic growth that is caused by the behaviour of the private sector. This again tends to reduce the difference between d_P and d_P^T via the impact on d_P^T. Consequently, fiscal policy reduces the amplitude of the cycles by suppressing the destabilising forces that stem from the endogenous changes in the desired margins of safety. This is also beneficial to the government sector itself. Since after some periods the fluctuations in economic growth decline, the same happens to the government balance (as a proportion of output) and the net government debt ratio. Therefore, although at a first place there might be some adverse developments in the fiscal performance, in the medium to long run fiscal prudence is safeguarded under the Godley-Minsky fiscal rule.

Interestingly enough, the simulations in Figure 6 indicate that the Godley-Minsky fiscal rule is not stabilising when θ is close to zero, i.e. when the target net debt ratio of the private sector does not change endogenously (or changes only slightly). The reason is that Figure 6 refers to a private sector that has a net creditor benchmark position. As explained in Section 2, in this case a higher (lower) growth rate tends to increase (decrease) the private sector's actual net debt ratio – this is exactly the opposite to what happens when the private sector has a net debtor benchmark position. Hence, when, for example, $d_P > d_P^T$ (i.e. the private sector does not spend enough), the increase in growth caused by the counter-cyclical government expenditures leads to a rise in d_P which does not facilitate the convergence of d_P towards d_P^T. This implies that when θ is close to zero, the counter-cyclical effects of the Godley-Minsky fiscal rule are not conducive to stability. This, however, does not hold when the private sector has a net debtor benchmark position ($d_P^B > 0$). In this case, the Goldey-Minsky fiscal rule is stabilising even when θ is close to zero.

<Insert Figure 6 here>

Figure 7 and Figure 8 provide an additional comparison of the performance of the Maastricht-type and the Godley-Minsky fiscal rules. Figure 7 shows how the macro system responds under the two different rules when the growth rate of exports goes down due to an exogenous shock (for example, because of a decline in the income of the foreign sector). In the case of the Maastricht-type fiscal rule, the induced decline in the growth rate of output increases the debt ratio of the government, leading to a reduction in government expenditures that reinforces the contractionary process. As illustrated in Figure 7 (I), this initiates a period of instability whereby cycles become much more severe as time passes. On the other hand, under the Godley-Minsky fiscal rule, the cycles are very mild (Figure 7 (II)). Thus, although the growth rate of output becomes permanently lower (due to the deterioration in the export performance) and the government debt-to-output ratio becomes, thus, slightly higher, the Godley-Minsky fiscal rule prevents the economy from entering into a period of instability.

<Insert Figure 7 here>

Figure 8 plots the adjustment of the macro system if the interest rate increases exogenously (for instance, because of a confidence crisis in the government bonds market). In our model a rise in the interest rate tends to increase public indebtedness, but its impact on the net debt ratio of the private sector depends on whether d_P^B is positive or negative. If $d_P^B > 0$, a higher interest rate tends to increase private sector's net debt ratio. If $d_P^B < 0$, a rise in the interest rate increases the net interest payments, so it tends to reduce private sector's net debt ratio. In Figure 8 we consider the more interesting case in which $d_P^B > 0$ (the results obtained when $d_P^B < 0$ are similar). The rise in the interest rate drives up the indebtedness of the private sector and thereby places downward pressures on the propensity to spend and the growth rate. It also causes an increase in the indebtedness of the government sector. Under the Maastricht-type fiscal rule, the rise in the government debt ratio leads to lower government expenditures. This does not help the economy recover from the fact that the private sector has reduced its spending.

On the contrary, the Godley-Minsky fiscal rule reacts to the increase in private indebtedness and not to the rise in public indebtedness. This means that fiscal authorities increase government expenditures, counterbalancing the decline in private spending. This stabilises macroeconomic activity and, as a result, all the macro variables. Hence, we overall have that the Maastricht-type fiscal rule is conducive to instability, while the Godley-Minsky fiscal rule helps the system stabilise after the interest rate shock. Note, though, that the Godley-Minsky fiscal rule is not able to change the fact that the new steady-state value of the government debt-to-output ratio is higher because of the higher interest rate.

<Insert Figure 8 here>

6. Conclusion

This paper developed a simple macrodynamic model that synthesises certain aspects of Godley's and Minsky's analytical frameworks. Using this model, it was first shown that Godley debt cycles are likely to arise as a result of the responsiveness of private sector's propensity to spend to divergences between the actual and the target net private debt ratio (the stock-flow norm). These cycles are generated when the private sector has a net debtor benchmark position. Instability emerges when the responsiveness of the propensity to spend is sufficiently high. This is a paradox of debt result: the more the private sector and its lenders attempt to put net private indebtedness under control, by adjusting private expenditures, the more the net private debt ratio destabilises. This result is associated with the economic growth consequences of this adjustment which have destabilising feedback effects on net private indebtedness.

Furthermore, the analysis indicated that when the target net private debt ratio is allowed to change endogenously via a Minsky mechanism an otherwise stable macro system can be transformed into an unstable one. This happens, in particular, when the expectations that determine the stock-flow norm are highly sensitive to the changes in economic performance. Godley-Minsky debt cycles can also be generated. These cycles are driven by the changes in euphoria and the perceptions of risk that affect Godley's stock-flow norm.

Lastly, the paper examined the implications of the developed framework for fiscal policy by investigating the (de)stabilising effects of a Maastricht-type fiscal rule and a Godley-Minsky fiscal rule. Simulation analysis illustrated that the Maastricht-type fiscal rule is destabilsing while the Godley-Minsky fiscal rule is stabilising. The paradox of debt appears to apply to the government sector: the more the fiscal authorities attempt to target a specific government debt ratio, by adjusting the government expenditures, the more this ratio destabilises. Moreover, it was shown that the Maastricht-

type fiscal rule increases the instability caused from adverse exogenous shocks that affect the interest rates or the export performance of an economy. Instead, the Godley-Minsky fiscal rule reduces significantly the instability generated from these shocks.

The model of this paper and the results presented above bring to the fore the importance of Godley's and Minsky's perspectives on the inherent instability of the macroeconomy, the generation of debt cycles and the stabilising role of fiscal policy. Based on these perspectives, the paper provided a new look at the dynamics of the modern macroeconomies in which the financial relationships between the private, the government and the foreign sector play a crucial role. An important line of research would be to combine the Godley-Minsky cycles produced here, which focus on aggregate sectoral debt, with the traditional Goodwin cycles that concentrate on the intra-private sector relationships and the role of income distribution.[22]

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Footnotes

[1] See, for example, Whalen (2008), Wolf (2008), Bezemer (2010) and Wray (2011, 2012).

[2] According to Godley *et al.* (2007) and Zezza (2009), the slight recession of 2001 was a first sign of the unsustainable processes in the US economy; a crisis was then prevented due to accommodative fiscal and monetary policies.

[3] The assumption that the interest rate is exogenous, and thus independent of the level of (private and government) indebtedness, seems quite restrictive. However, this assumption has been adopted because one of the purposes of this model is to show that indebtedness can be a source of cycles and instability even when it has no direct impact on the interest rates. Note, though, that the adverse effects that high indebtedness can have on the new debt inflow (by causing a rise in the interest rates) are implicitly incorporated in equations (8) and (10A) below.

[4] The balances of the three sectors as a proportion of output are presented in Appendix B2.

[5] For an analysis of this approach see, for example, Godley and Cripps (1983), Zezza (2009), Dos Santos and Macedo e Silva (2010), Brecht *et al.* (2012) and Wray (2012). Interestingly, Minsky also used some aspects of the financial balances approach (see, e.g., Minsky, 1982, pp. 5-6).

[6] For simplicity, in equations (1) and (3) we use the private sector's income after taxes, but before the interest payments.

[7] See also Martin (2012) and Shaikh (2012).

[8] Interestingly, formula (8) shares some similarities with the recent macroeconomic analysis of Koo (2013) about what he calls a 'balance sheet recession'. In this analysis Koo makes a distinction between periods in which the private sector maximises profits ('Yang phases') and periods in which the private sector minimises its debt ('Yin phases'). In our model, the reduction in the private sector's propensity to spend when $d_P^T < d_P$ resembles a 'Yin phase' à la Koo.

[9] See also Kregel (1997), Vercelli (2011) and Lavoie (2014, p. 446).

[10] Our formulation draws on Nikolaidi (2014) who assumes a positive impact of investment on the desired margins of safety of firms and banks.

[11] For recent Minskyan models that incorporate explicitly this role see Chiarella and Di Guilmi (2011), Passarella (2012) and Ryoo (2013A).

[12] For a Minsky model that formalises cycles with different time horizons see Ryoo (2013A). His model relies on a Harrodian perspective and concentrates on the interactions between households, firms and commercial banks.

[13] For a fiscal rule that relies on a similar rationale see Nikolaidi (2014). Interestingly, the Godley-Minsky fiscal rule is in line with Koo's (2013) proposition that during 'Yin phases' fiscal expansion is necessary in order to avoid deep recessions.

[14] When equation (10B) is utilised instead of (10A), the whole macro system is a 4D system since d_G has no feedback effects on the other state variables.

[15] In the simulations the function $\Xi\left(d_P^T - d_P\right)$ in equation (8) takes the form $\xi_1 \, tanh[\, \xi_2 (\, d_P^T - d_P \,)]$. Likewise, the function $Z\left(g_Y - g_{Y0}\right)$ in equation (9) takes the form $\zeta_1 \, tanh[\, \zeta_2 (\, g_Y - g_{Y0} \,)]$. The MATLAB codes for the simulations of this paper draw on Nikolaidi (2014). The codes are available upon request.

[16] Stability also arises in the marginal case in which the private sector is in a zero net debt benchmark position (i.e. $d_P^B = 0$).

[17] Recent works that apply the limit cycle analysis to economic dynamic systems include Assous and Dutt (2013), Datta (2014), von Arnim and Barrales (2015), Ryoo (2016) and Stockhammer and Michell (2017).

[18] For the paradox of debt in the case of firms see, for example, Lavoie (1995), Passarella (2012) and Ryoo (2013B).

[19] For similar figures that show how changes in specific parameters affect the stability properties of dynamic systems, see Chiarella *et al.* (2012) and Nikolaidi (2014).

[20] For the destabilising effects of Maastricht-type fiscal rules see also Charpe *et al.* (2011, ch. 9).

[21] The potential application of the paradox of debt to the government sector has been briefly pointed out by Lavoie (2014, p. 19).

[22] For the presentation of the Goodwin cycles and the combination of Goodwin's model with Minskyan dynamics see Keen (1995), Grasselli and Costa Lima (2012), Sordi and Vercelli (2014) and Stockhammer and Michell (2017).

Appendix A: Glossary of variables and parameters

Variable/parameter	Definition
B_F	Balance of the foreign sector
b_F	Foreign balance-to-output ratio
B_G	Balance of the government sector
b_G	Government balance-to-output ratio
B_P	Balance of the private sector
b_P	Private balance-to-output ratio
C	Consumption
D_P	Net private debt
d_P	Net private debt-to-income ratio
d_P^B	Benchmark net private debt-to-income ratio
d_P^T	Target net private debt-to-income ratio
D_G	Net government debt
d_G	Net government debt-to-output ratio
d_G^T	Target net government debt-to-output ratio
I	Private sector's investment
G	Primary government expenditures
g	Primary government expenditures-to-output ratio
g_X	Growth rate of exports
g_Y	Growth rate of output
g_{Y0}	Normal growth rate of output
m	Imports-to-output ratio
M	Imports
p	Private sector's propensity to spend
p^B	Private sector's benchmark propensity to spend
P	Private expenditures (consumption plus investment)
r	Interest rate
S	Private sector's saving
T	Taxes
Y	Output
Y_P	Private sectror's discposable income
X	Exports
ζ_1	Parameter capturing the responsiveness of the private sector's acceptable target debt ratio to the gap between the actual and the benchmark growth rate
ζ_2	Parameter capturing the responsiveness of the private sector's acceptable target debt ratio to the gap between the actual and the benchmark growth rate
θ	Parameter capturing the adjustment of the private sector's target net debt ratio towards the acceptable target debt ratio (the latter is affected by $(g_Y - g_{Y0})$)
κ_1	Reaction parameter in the Godley-Minsky fiscal rule
κ_2	Reaction parameter in the Godley-Minsky fiscal rule
λ	Parameter capturing the adjustment of the private sector's propensity to spend towards the desired propensity to spend (the latter is affected by $(d_P^T - d_P)$)
μ	Reaction parameter in the Maastricht-type fiscal rule
ξ_1	Responsiveness of the private sector's desired propensity to spend to the gap between the target and the actual debt ratio
ξ_2	Responsiveness of the private sector's desired propensity to spend to the gap between the target and the actual debt ratio
τ	Taxes-to-output ratio
Notational convention	
$\dot{x}$	Time derivative of variable x, i.e. $\dot{x} \equiv dx/dt$

Note: In the simulations the function $\Xi\left(d_P^T - d_P\right)$ *in equation (8) takes the form* $\xi_1 \tanh[\xi_2(d_P^T - d_P)]$ *and the function* $Z\left(g_Y - g_{Y0}\right)$ *in equation (9) takes the form* $\zeta_1 \tanh[\zeta_2(g_Y - g_{Y0})]$ *.*

Appendix B: Additional equations of the model and algebraic details

B1. Identities of the model

$$Y = C + I + G + X - M \tag{B1}$$
$$Y_P = Y - T - rD_P \tag{B2}$$
$$S = Y_P - C \tag{B3}$$
$$\dot{D}_P = -B_P = P - Y_P \tag{B4}$$
$$\dot{D}_G = -B_G = G - T + rD_G \tag{B5}$$
$$\dot{D}_F = -B_F = X - M - r(D_P + D_G) \tag{B6}$$
$$D_P + D_G + D_F = 0 \tag{B7}$$
$$\dot{D}_P + \dot{D}_G + \dot{D}_F = -(B_P + B_G + B_F) = 0 \tag{B8}$$

Note that $P = C + I$.

B2. The balances of the three sectors as a proportion of output

Combining equations (B2), (B4)-(B6) and (1)-(6), we get:

$$b_P = \frac{B_P}{Y} = (1 - p - rd_P)(1 - \tau) \tag{B9}$$
$$b_G = \frac{B_G}{Y} = \tau - g - rd_G \tag{B10}$$
$$b_F = \frac{B_F}{Y} = m - \frac{X}{Y} + r\left[d_P(1-\tau) + d_G\right] \tag{B11}$$

Note that $b_P + b_G + b_F = 0$. This can be easily shown by substituting the ratio X/Y from equation (B13) into equation (B11).

B3. The growth rate of output

Combining equations (B1), (1), (2), (5) and (6), we get:

$$Y = pY(1-\tau) + gY + X - mY \tag{B12}$$

Solving (B12) for Y, yields:

$$Y = \frac{X}{1 + m - p(1-\tau) - g} \tag{B13}$$

The denominator in equation (B13) must be positive (i.e. $1 + m - p(1-\tau) - g > 0$) to ensure goods market stability. Differentiating equation (B13) with respect to time and dividing through by Y, gives the growth rate of the economy:

$$\frac{\dot{Y}}{Y} = g_Y = g_X + \frac{\dot{p}(1-\tau) + \dot{g}}{1 + m - p(1-\tau) - g} \tag{B14}$$

B4. The steady state of the 5D (4D) system

The system defined by equations (12), (8), (9), (13) and (10A) or (10B) has a unique steady state. The steady-state values (denoted by the subscript $_0$) of the state variables of the system are:

$$d_{P0} = d_{P0}^T = d_P^B$$
$$d_{G0} = d_G^T$$
$$p_0 = 1 + (g_X - r)d_P^B = p^B$$
$$g_0 = \tau + (g_X - r)d_G^T$$

Note that d_P^B and d_G^T are parameters. At the steady state we also have that:

$$g_{Y0} = g_X$$
$$b_{P0} = -g_X d_P^B (1-\tau)$$
$$b_{G0} = -g_X d_G^T$$
$$b_{F0} = g_X [d_P^B (1-\tau) + d_G^T]$$

B5. Stability analysis of $\dot{d}_P$ and $\dot{d}_G$ when economic growth is exogenous

When economic growth is exogenous, p, g and d_P^T are at their steady-state values, and thereby $\dot{p} = \dot{g} = 0$. Hence, according to equation (11), economic growth is equal to the growth rate of exports (i.e. $g_Y = g_X$). Under these conditions, from equations (12) and (13) we get:

$$\partial \dot{d}_P / \partial d_P = r - g_X$$
$$\partial \dot{d}_G / \partial d_G = r - g_X$$

Stability requires that $\partial \dot{d}_P / \partial d_P < 0$ and $\partial \dot{d}_G / \partial d_G < 0$. This holds when $g_X > r$. As explained in Section 2, this condition is assumed to be always true throughout the paper.

Appendix C: Parameter values in the simulations

Parameter	Fig. 1a 2D system	Fig. 1b 2D system	Fig. 2 2D system	Fig. 3 3D system	Fig. 4 3D system	Fig. 5 5D system	Fig. 6 4D system	Fig. 7 5D (4D) system	Fig. 8 5D (4D) system	Justification
r	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.027	Close to 0.026, which is the unweighted average of the long-term real interest rate for 35 EU and non-EU countries over the period 1961-2014 (source: AMECO; code: ILRV).
g_X	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.026	0.03	The baseline value is slightly higher than the interest rate, ensuring stability when the laws of motion of debt ratios are examined in isolation (see Section 2).
τ	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	Close to 0.34, which is the unweighted average of the total tax burden (excluding imputed social contributions), as a proportion of GDP, for 34 EU and non-EU countries over the period 1975-2014 (source: AMECO; code: UTAT).
m	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	Close to 0.41 which is the unweighted average of the imports-to-GDP ratio for 43 EU and non-EU countries over the period 1960-2014 (source: AMECO; codes: UMGS, UVGD).
d_G^T	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	Equal to 0.6 minus 0.3; 0.6 is the maximum accepted value of the gross public debt ratio according to the Maastricht criteria; 0.3 is the average difference between gross and net goveernment debt ratio in 27 advanced countries over the period 2001-2012 (source: IMF, World Economic Outlook Database, April 2015 edition).
d_P^B	0.2	0.2	0.2	-0.2	-0.2	-0.2	-0.2	-0.2	0.2	Has been chosen such that the steady-state private balance (b_{P0}) is slightly lower/higher than 0.
λ	0.005	0.1	0.1	0.02	0.02	0.02	0.02	0.02	0.01	Selected such that stability/instability is produced in the 2D subsystem
λ^*	0.0163	0.0163	0.0163	-	-	-	-	-	0.0163	Estimated based on the formula presented in Appendix D
ξ_1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	Selected from a reasonable range of values
ξ_2	20	20	20	20	20	20	20	20	20	Selected from a reasonable range of values
θ	0	0	0	-	0.1	-	-	0.1	0.1	Selected such that stability/instability is produced in the 3D subsystem
θ^*	-	-	-	0.0503	0.0503	0.0503	0.0503	0.0503	-	Estimated based on the formula presented in Appendix E
ζ_1	0	0	0	0.4	0.4	0.4	0.4	0.4	0.4	Selected from a reasonable range of values
ζ_2	0	0	0	20	20	20	20	20	20	Selected from a reasonable range of values
μ	0	0	0	0	0	0.05	0	0.05 (0)	0.05 (0)	Selected from a reasonable range of values
$\varkappa_1$	0	0	0	0	0	0	0.1	0 (0.1)	0 (0.1)	Selected from a reasonable range of values
$\varkappa_2$	0	0	0	0	0	0	0.02	0 (0.02)	0 (0.02)	Selected from a reasonable range of values

Appendix D: Stability analysis of the 2D subsystem

Suppose that the variables g and d_P^T are kept at their steady-state values (i.e. $g = g_0$ and $d_P^T = d_P^B$). Then, for the 2D system consisting of equations (12) and (8) it can be proved that:

(a) If $d_P^B \leq 0$, the steady state of the system is locally stable.

(b) If $d_P^B > 0$, for a sufficiently high $\left|\Xi_{d_P}\right|$ (i.e. $\left|\Xi_{d_P}\right| > \xi^* > 0$) there exists a parameter value $\lambda^* > 0$ that satisfies the following properties:

(i) For all $\lambda \in (0, \lambda^*)$ the steady state of the system is locally stable.

(ii) For all $\lambda \in (\lambda^*, +\infty)$ the steady state of the system is locally unstable and the trajectories of $p(t)$ and $d_P(t)$ converge to a limit cycle.

(iii) At $\lambda = \lambda^*$ the system undergoes a Hopf bifurcation (in other words, there exists a limit cycle around the steady state for some range of the parameter value λ which is sufficiently close to λ^*).

In order to prove the above proposition, we need to estimate the Jacobian matrix of the system. This matrix, evaluated at the steady state, is:

$$J_{2D} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix},$$

where:

$$J_{11} = \partial \dot{d}_P / \partial d_P = r - g_X - \frac{d_P^B (1-\tau)\lambda \Xi_{d_P}}{1 + m - p_0(1-\tau) - g_0}$$
$$J_{12} = \partial \dot{d}_P / \partial p = 1 + \frac{d_P^B (1-\tau)\lambda}{1 + m - p_0(1-\tau) - g_0}$$
$$J_{21} = \partial \dot{p} / \partial d_P = \lambda \Xi_{d_P} < 0$$
$$J_{22} = \partial \dot{p} / \partial p = -\lambda < 0$$

We have:

$$Tr(J_{2D}) = J_{11} + J_{22} = r - g_X - \frac{d_P^B(1-\tau)\lambda\Xi_{d_P}}{1+m-p_0(1-\tau)-g_0} - \lambda$$
$$Det(J_{2D}) = J_{11}J_{22} - J_{21}J_{12} = -\lambda(r-g_X) - \lambda\Xi_{d_P} > 0$$

Proof of (a):

If $d_P^B < 0$, then $Tr(J_{2D}) < 0$; recall that $g_X > r$ (see Appendix B5) and $1+m-p_0(1-\tau)-g_0 > 0$ (see Appendix B3). Since $Det(J_{2D})$ is always positive, the steady state is locally stable. □

Proof of (b-i):

If $d_P^B > 0$, then $Tr(J_{2D}) < 0$ for sufficiently low values of λ. By setting $Tr(J_{2D}) = 0$, we can find the critical value for λ below which $Tr(J_{2D}) < 0$:

$$\lambda^* = \frac{-(r-g_X)}{\dfrac{-\Xi_{d_P} d_P^B(1-\tau)}{1+m-p_0(1-\tau)-g_0} - 1}$$

We assume that $|\Xi_{d_P}|$ is sufficiently high such that $|\Xi_{d_P}| > \xi^* = (1+m-p_0(1-\tau)-g_0)/(1-\tau)d_P^B$. This ensures $\lambda^* > 0$ since $g_X > r$. The steady state is locally stable when $\lambda < \lambda^*$. □

Proof of (b-ii):

If $d_P^B > 0$ and $\lambda > \lambda^*$, we have $Tr(J_{2D}) > 0$. Hence, since $Det(J_{2D})$ is always positive, the steady state is locally unstable.

It can be proved that the trajectories of $p(t)$ and $d_P(t)$ are bounded. To prove that, we follow closely the methods used by Ryoo (2013A, 2016). We write the endogenous variables as a function of time. Due to the boundedness of function Ξ in equation (8), in period s we have:

$$\underline{\lambda} < \Xi\left(d_P^T - d_P(s)\right) < \overline{\lambda}$$

where $\underline{\lambda}$ is the lower bound and $\overline{\lambda}$ is the upper bound of function Ξ.

Substituting from equation (8):

$$\underline{\lambda} < \frac{\dot{p}(s)}{\lambda} - p^B + p(s) < \overline{\lambda}$$

Multiplying by $e^{\lambda s}$, integrating over $[0,t]$ and re-arranging, we have:

$$\lambda\left(\underline{\lambda} + p^B\right)\int_0^t e^{\lambda s} ds < \int_0^t e^{\lambda s}\dot{p}(s) ds + \lambda\int_0^t e^{\lambda s} p(s) ds < \lambda\left(\overline{\lambda} + p^B\right)\int_0^t e^{\lambda s} ds$$

After some tedious algebra, we obtain:

$$\left(\underline{\lambda} + p^B\right)\left(e^{\lambda t} - 1\right) < e^{\lambda t} p(t) - p(0) < \left(\overline{\lambda} + p^B\right)\left(e^{\lambda t} - 1\right)$$

Multiplying by $e^{-\lambda t}$:

$$\left(\underline{\lambda} + p^B\right)\left(1 - e^{-\lambda t}\right) + p(0)e^{-\lambda t} < p(t) < \left(\overline{\lambda} + p^B\right)\left(1 - e^{-\lambda t}\right) + p(0)e^{-\lambda t}$$

For all $t \in [0, \infty]$, $0 < e^{-\lambda t} < 1$. Hence, the above inequality can be re-written as:

$$p_{min} = min\{ p(0), \underline{\lambda} + p^B \} \leq p(t) \leq max\{ p(0), \overline{\lambda} + p^B \} = p_{max}$$

This implies that $p(t)$ is bounded from above and below.

To prove the boundedness of $d_P(t)$, we use equation (12). For a given value of p (i.e. $\dot{p} = 0$), there is a specific value of $d_P = \tilde{d}_P(p)$ that satisfies the condition:

$$\dot{d}_P = p - 1 + (r - g_X)\tilde{d}_P(p) = 0$$

Hence, when p is given, the steady-state debt ratio is:

$$\tilde{d}_P(p) = \frac{1-p}{r - g_X}$$

Since $r - g_X < 0$, $\tilde{d}_P{}'(p) > 0$ (i.e. $\tilde{d}_P(p(t))$ is increasing in $p(t)$). We have also shown that the steady-state debt ratio is stable (see Appendix B5).

Since $p_{\min} \leq p(t) \leq p_{\max}$, $\tilde{d}_P{}'(p) > 0$ implies $\tilde{d}_P(p_{\min}) \leq \tilde{d}_P(p(t)) \leq \tilde{d}_P(p_{\max})$. We claim that $d_P(t)$ is bounded such that:

$$d_{P\min} = \min\{d_P(0), \tilde{d}_P(p_{\min})\} \leq d_P(t) \leq \max\{d_P(0), \tilde{d}_P(p_{\max})\} = d_{P\max}$$

To prove that, consider the following cases:

(i) $d_P(s) < \tilde{d}_P(p_{\min})$

(ii) $d_P(s) > \tilde{d}_P(p_{\max})$

(iii) $\tilde{d}_P(p_{\min}) \leq d_P(s) \leq \tilde{d}_P(p_{\max})$

In Case (i) we have $d_P(s) < \tilde{d}_P(p(s))$ since $\tilde{d}_P(p_{\min}) \leq \tilde{d}_P(p(s))$. Because $d_P(p)$ is stable, we therefore get $\dot{d}_P(s) > 0$. In Case (ii), we have $d_P(s) > \tilde{d}_P(p(s))$ since $\tilde{d}_P(p_{\max}) \geq \tilde{d}_P(p(s))$. Because $d_P(p)$ is stable, we therefore get $\dot{d}_P(s) < 0$. Hence, we overall have that any trajectory that starts outside the interval $[\tilde{d}_P(p_{\min}), \tilde{d}_P(p_{\max})]$ is attracted to this interval.

In Case (iii) we claim that any trajectory that starts inside the interval cannot escape from it. To prove this suppose that it can. Then, there must exist an $s' > s$ such that $d_P(s') \notin [\tilde{d}_P(p_{\min}), \tilde{d}_P(p_{\max})]$. Without loss of generality, let us consider the case in which $d_P(s') < \tilde{d}_P(p_{\min})$. Because of the continuity of $d_P(t)$ there must also exist an s'' such that $s' > s'' > s$ and $d_P(s') < d_P(s'') < \tilde{d}_P(p_{\min})$. The mean value theorem implies that there exists an s''' such that $s' > s''' > s'' > s$ and $\dot{d}_P(s''') = \frac{d_P(s') - d_P(s'')}{s' - s''} < 0$. This, however, is not consistent with the fact that $d_P(s') > 0$ whenever $d_P(t) < \tilde{d}_P(p_{\min})$.

Overall, it has been shown that the trajectories of $p(t)$ and $d_P(t)$ are bounded. Since the steady state is unique and locally unstable, the Poincarè-Bendixson theorem implies that these trajectories converge to a limit cycle (for a similar use of the Poincarè-Bendixson theorem see, for example, Nikaido, 1996, ch. 12, Datta, 2014 and Ryoo, 2016). □

Proof of (b-iii):

At $\lambda = \lambda^*$, $Det(J_{2D}) > 0$ and $Tr(J_{2D}) = 0$. This implies that the characteristic equation of the system has a pair of pure imaginary roots. Furthermore, we have:

$$\frac{d}{d\lambda}\left(Tr(\lambda)\right)\Big|_{\lambda=\lambda^*} = \frac{d}{d\lambda}\left(r - g_X - \frac{d_P^B(1-\tau)\lambda\Xi_{d_P}}{1+m-p_0(1-\tau)-g_0} - \lambda\right)\Bigg|_{\lambda=\lambda^*} = -\frac{d_P^B(1-\tau)\Xi_{d_P}}{1+m-p_0(1-\tau)-g_0} - 1$$

Since $\left|\Xi_{d_P}\right| > \xi^* = (1+m-p_0(1-\tau)-g_0)/(1-\tau)d_P^B$, we have $\frac{d}{d\lambda}\left(Tr(\lambda)\right)\Big|_{\lambda=\lambda^*} > 0$. This means that at $\lambda = \lambda^*$ the real part of the imaginary roots is not stationary with respect to changes in λ (note that the trace of the Jacobian matrix represents the real part of the roots). Therefore, the conditions for the existence of a Hopf bifurcation point are satisfied. □

Appendix E: Stability analysis of the 3D subsystem

Suppose that the steady state of the 2D subsystem consisting of equations (12) and (8) is stable and the variable g is kept at its steady-state value (i.e. $g = g_0$). Then, for the 3D system consisting of equations (12), (8) and (9) it can be proved that for a sufficiently high Z_{g_Y} (i.e. $Z_{g_Y} > \zeta^* > 0$) there exists a parameter value $\theta^* > 0$ which satisfies the following properties:

(i) For all $\theta \in (0, \theta^*)$ the steady state of the system is locally stable.

(ii) For all $\theta \in (\theta^*, +\infty)$ the steady state of the system is locally unstable and the trajectories of $p(t)$, $d_P(t)$ and $d_P^T(t)$ exhibit cyclical fluctuations.

(iii) At $\theta = \theta^*$ the system undergoes a Hopf bifurcation (in other words, there exists a limit cycle around the steady state for some range of the parameter value θ which is sufficiently close to θ^*).

In order to prove the above proposition, we need to estimate the Jacobian matrix of the system. This matrix, evaluated at the steady state, is:

$$J_{3D} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix},$$

where:

$$J_{11} = \partial \dot{d}_P / \partial d_P = r - g_X - \frac{d_P^B (1-\tau)\lambda \Xi_{d_P}}{1 + m - p_0(1-\tau) - g_0}$$

$$J_{12} = \partial \dot{d}_P / \partial p = 1 + \frac{d_P^B (1-\tau)\lambda}{1 + m - p_0(1-\tau) - g_0}$$

$$J_{13} = \partial \dot{d}_P / \partial d_P^T = -\frac{d_P^B (1-\tau)\lambda \Xi_{d_P^T}}{1 + m - p_0(1-\tau) - g_0}$$

$$J_{21} = \partial \dot{p} / \partial d_P = \lambda \Xi_{d_P} < 0$$

$$J_{22} = \partial \dot{p} / \partial p = -\lambda < 0$$

$$J_{23} = \partial \dot{p} / \partial d_P^T = \lambda \Xi_{d_P^T} > 0$$

$$J_{31} = \partial \dot{d}_P^T / \partial d_P = \frac{\theta Z_{g_Y} \lambda \Xi_{d_P} (1-\tau)}{1+m-p_0(1-\tau)-g_0} < 0$$

$$J_{32} = \partial \dot{d}_P^T / \partial p = -\frac{\theta Z_{g_Y} \lambda (1-\tau)}{1+m-p_0(1-\tau)-g_0} < 0$$

$$J_{33} = \partial \dot{d}_P^T / \partial d_P^T = \theta \left(\frac{Z_{g_Y} \lambda \Xi_{d_p^T} (1-\tau)}{1+m-p_0(1-\tau)-g_0} - 1 \right)$$

The characteristic equation of the system is:

$$\Phi(d) = d^3 + a_1 d^2 + a_2 d + a_3 = 0$$

We have:

(i) $a_1 = -(J_{11} + J_{22} + J_{33}) = -Tr(J_{2D}) - \left(\frac{Z_{g_Y} \lambda \Xi_{d_p^T} (1-\tau)}{1+m-p_0(1-\tau)-g_0} - 1 \right) \theta = -Tr(J_{2D}) + \Omega_1 \theta$,

where $Tr(J_{2D}) < 0$ and $\Omega_1 = -\left(\frac{Z_{g_Y} \lambda \Xi_{d_p^T} (1-\tau)}{1+m-p_0(1-\tau)-g_0} - 1 \right)$.

(ii) $a_2 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}$ or

$$a_2 = Det(J_{2D}) + \left(\frac{Z_{g_Y} \lambda \Xi_{d_p^T} (1-\tau)(r-g_X)}{1+m-p_0(1-\tau)-g_0} - Tr(J_{2D}) \right) \theta = Det(J_{2D}) + \Omega_2 \theta,$$

where $Det(J_{2D}) > 0$ and $\Omega_2 = \frac{Z_{g_Y} \lambda \Xi_{d_p^T} (1-\tau)(r-g_X)}{1+m-p_0(1-\tau)-g_0} - Tr(J_{2D})$.

(iii) $a_3 = -Det(J_{3D}) = -\begin{vmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{vmatrix} = Det(J_{2D})\theta$

where $Det(J_{2D}) > 0$.

(iv) $b = a_1 a_2 - a_3 = \left(-Tr\left(J_{2D}\right) + \Omega_1\theta\right)\left(Det\left(J_{2D}\right) + \Omega_2\theta\right) - Det\left(J_{2D}\right)\theta$.

We assume that Z_{g_Y} is sufficiently high such that $Z_{g_Y} > \zeta^* = \dfrac{1 + m - p_0(1-\tau) - g_0}{\lambda \Xi_{d_P^T}(1-\tau)}$. Hence, $\Omega_1 < 0$ and a_1 is thereby a negative function of θ. Setting $a_1 = 0$ and solving for θ, we obtain $\theta^{a_1} = Tr\left(J_{2D}\right)/\Omega_1 > 0$. Hence, for $\theta < \theta^{a_1}$, a_1 is positive. Otherwise, a_1 is negative.

The sign of a_2 is also ambiguous. We have the following cases:

- *Case I*: $\Omega_2 > 0$. a_2 is always positive.
- *Case II*: $\Omega_2 < 0$. a_2 is a negative function of θ. Setting $a_2 = 0$ and solving for θ, we obtain $\theta^{a_2} = -Det\left(J_{2D}\right)/\Omega_2 > 0$. Hence, for $\theta < \theta^{a_2}$, a_2 is positive. Otherwise, a_2 is negative.

$a_3 > 0$ since $Det\left(J_{2D}\right) > 0$.

The expression for b can be written as:

$$b = A\theta^2 + B\theta + \Gamma$$

where:

$$A = \Omega_1 \Omega_2$$

$$B = \left(-Tr\left(J_{2D}\right)\Omega_2 + (\Omega_1 - 1)Det\left(J_{2D}\right)\right)$$

$$\Gamma = -Tr\left(J_{2D}\right)Det\left(J_{2D}\right)$$

The quadratic equation $b = 0$ has two roots:

$$\theta^{b_1} = \frac{-B + \sqrt{B^2 - 4A\Gamma}}{2A}$$

$$\theta^{b_2} = \frac{-B - \sqrt{B^2 - 4A\Gamma}}{2A}$$

It can be easily proved that $B^2 - 4A\Gamma > 0$ both in Case I and Case II (the proof is available upon request). Hence, the two roots are always real. The quadratic equation can therefore be expressed as:

$$b = \Omega_1\Omega_2(\theta - \theta^{b_1})(\theta - \theta^{b_2}) = 0$$

According to Vieta's formulas, $\theta^{b_1} + \theta^{b_2} = -B/A$ and $\theta^{b_1}\theta^{b_2} = \Gamma / A$. We always have $\Gamma > 0$.

In Case I, $A < 0$. Hence, $\theta^{b_1}\theta^{b_2} < 0$, which means that θ^{b_1} and θ^{b_2} are of opposite sign and $\theta^{b_2} > \theta^{b_1}$. It follows that $b > 0$ for all $\theta \in (\theta^{b_1}, \theta^{b_2})$ and $b < 0$ for all $\theta \in (-\propto, \theta^{b_1}) \cup (\theta^{b_2} + \propto)$. It can also be proved that $\theta^{b_2} < \theta^{a_1}$ (the proof is available upon request).

In Case II, $A > 0$ and $B < 0$. Hence, $\theta^{b_1} + \theta^{b_2} > 0$ and $\theta^{b_1}\theta^{b_2} > 0$, implying that both θ^{b_1} and θ^{b_2} are positive. Since $\theta^{b_1} > \theta^{b_2}$, it follows that $b > 0$ for all $\theta \in (-\propto, \theta^{b_2}) \cup (\theta^{b_1} + \propto)$ and $b < 0$ for all $\theta \in (\theta^{b_2}, \theta^{b_1})$. It can also be proved that $\theta^{b_2} < \theta^{a_1} < \theta^{b_1}$ and $\theta^{b_2} < \theta^{a_2} < \theta^{b_1}$ (the proof is available upon request).

Proof of (i):

Suppose $\theta^* = \theta^{b_2}$. If $\theta < \theta^*$, then $a_1, a_2, a_3, b > 0$. Therefore, according to the Routh-Hurwitz conditions, the steady state of the 3D subsystem is locally stable. □

Proof of (ii):

Suppose $\theta^* = \theta^{b_2}$. If $\theta > \theta^*$, then in Case I $b < 0$. In Case II we have $b < 0$ when $\theta < \theta^{b_1}$; also, $b \geq 0$ and $a_1, a_2 < 0$ when $\theta \geq \theta^{b_1}$. Therefore, at least one of the Routh-Hurwitz conditions is always violated and, hence, the steady state of the 3D subsystem is locally unstable.

In Appendix D is has been shown that the trajectories of $p(t)$ and $d_P(t)$ are bounded. Since equation (9) has exactly the same structure as equation (8), the same method as the one used in Appendix D to prove the boundedness of $p(t)$ can be employed to prove the boundedness of $d_P^T(t)$.

Since the trajectories of the 3D subsystem are bounded and its steady state is unique and locally unstable, the system exhibits cyclical fluctuations (see also Ryoo, 2013A, 2016). □

Proof of (iii):

Suppose $\theta^* = \theta^{b_2}$. At $\theta = \theta^*$, we have $a_1, a_2, a_3 > 0$ and $b = 0$. This implies that the characteristic equation has a pair of pure imaginary roots and a negative real root. Therefore, to prove that θ^* is a Hopf bifurcation point, it suffices to show that $\left.\frac{\partial \Gamma(\theta)}{\partial \theta}\right|_{\theta=\theta^*} \neq 0$, where Γ is the real part of the two imaginary roots. Asada and Semmler (1995) have proved that the condition $\left.\frac{\partial \Gamma(\theta)}{\partial \theta}\right|_{\theta=\theta^*} \neq 0$ is equivalent to the condition $\left.\frac{\partial b(\theta)}{\partial \theta}\right|_{\theta=\theta^*} \neq 0$ (see also Yoshida and Asada, 2007).

We have:

$$\frac{\partial b(\theta)}{\partial \theta} = 2\Omega_1\Omega_2\theta - Tr(J_{2D})\Omega_2 + \Omega_1 Det(J_{2D}) - Det(J_{2D})$$

Substituting $\theta = \theta^*$ into the above expression we get:

$$\left.\frac{\partial b(\theta)}{\partial \theta}\right|_{\theta=\theta^*} = -\sqrt{\left(-Tr(J_{2D})\Omega_2 + \Omega_1 Det(J_{2D}) - Det(J_{2D})\right)^2 + 4\Omega_1\Omega_2 Tr(J_{2D}) Det(J_{2D})} = -\sqrt{B^2 - 4A\Gamma} < 0 .$$

□

Figures and Tables

Table 1. Transactions matrix

	Private sector		Government sector	Foreign sector	Total
	Current	Capital			
Primary gov. expenditures	$+G$		$-G$		0
Taxes	$-T$		$+T$		0
Exports	$+X$			$-X$	0
Imports	$-M$			$+M$	0
Private sector's investment	$+I$	$-I$			0
Private sector's saving	$-S$	$+S$			0
Interest	$-rD_P$		$-rD_G$	$-rD_F$	0
Change in net debt		$+\dot{D}_P$	$+\dot{D}_G$	$+\dot{D}_F$	0
Total	0	0	0	0	0

Note: Plus signs indicate inflows/sources of funds and negative signs denote outflows/uses of funds.

Table 2. Balance sheet matrix

	Private sector	Government sector	Foreign sector	Total
Net debt	$+D_P$	$+D_G$	$+D_F$	0

Table 3. Stability properties of the 2D and 3D systems

	2D system	
	$d_P{}^B > 0$	$d_P{}^B \leq 0$
$\lambda < \lambda^*$	Stability	Stability
$\lambda \geq \lambda^*$	Instability (cycles)	Stability

	3D system	
	$d_P{}^B > 0$ & $\lambda < \lambda^*$	$d_P{}^B \leq 0$
$\theta < \theta^*$	Stability	Stability
$\theta \geq \theta^*$	Instability (cycles)	Instability (cycles)

Note: Analytical details and proofs are provided in Appendix D and Appendix E.

Fig. 1a. Dynamic adjustments of the 2D system to a 5% negative shock in the net private debt-to-income ratio; private sector in a net debtor benchmark position ($d_P^B > 0$); $\lambda = 0.005 < \lambda^* \approx 0.016$.

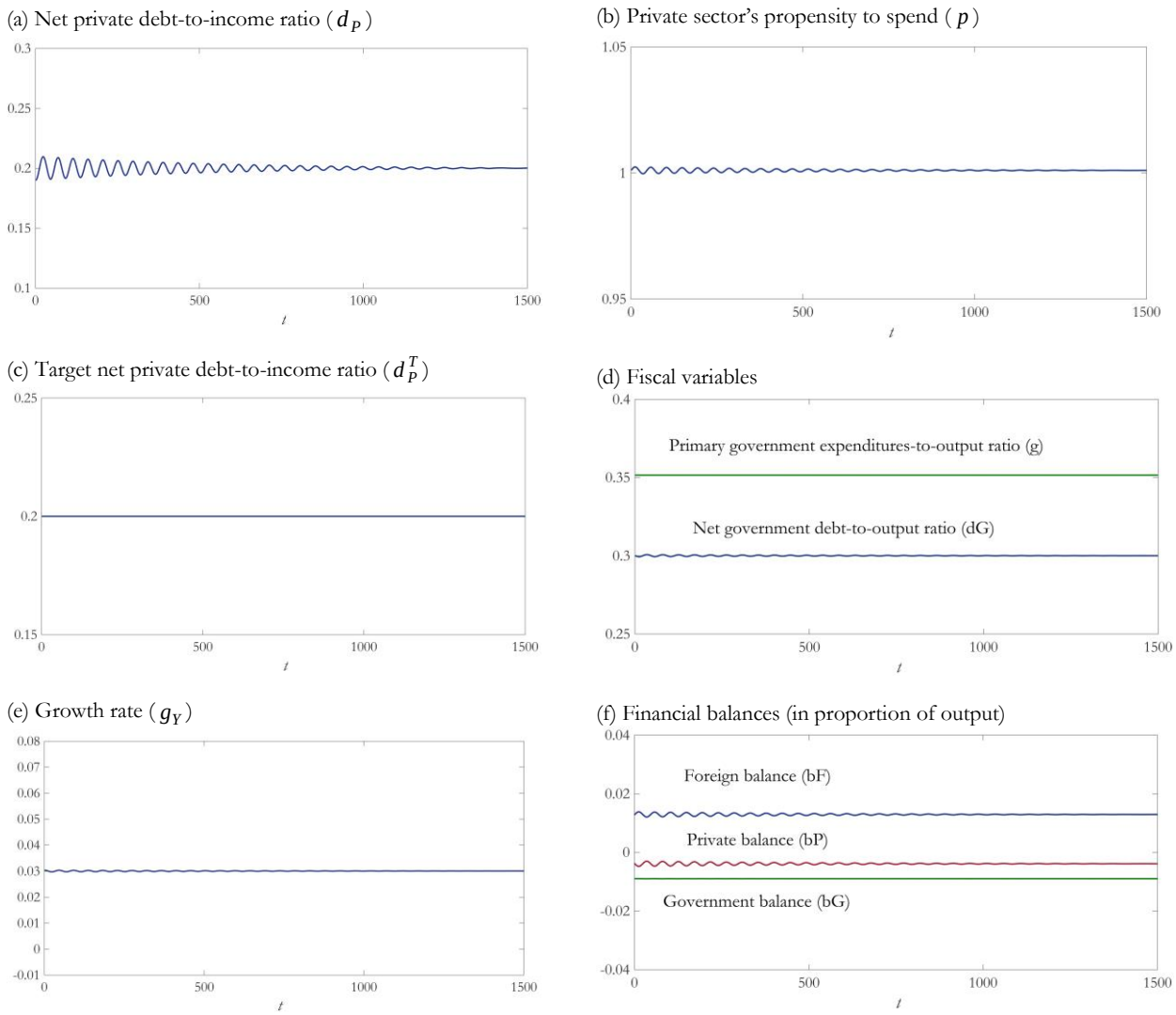


Fig. 1b. Dynamic adjustments of the 2D system to a 5% negative shock in the net private debt-to-income ratio; private sector in a net debtor benchmark position ($d_P^B > 0$); $\lambda = 0.1 > \lambda^* \approx 0.016$.

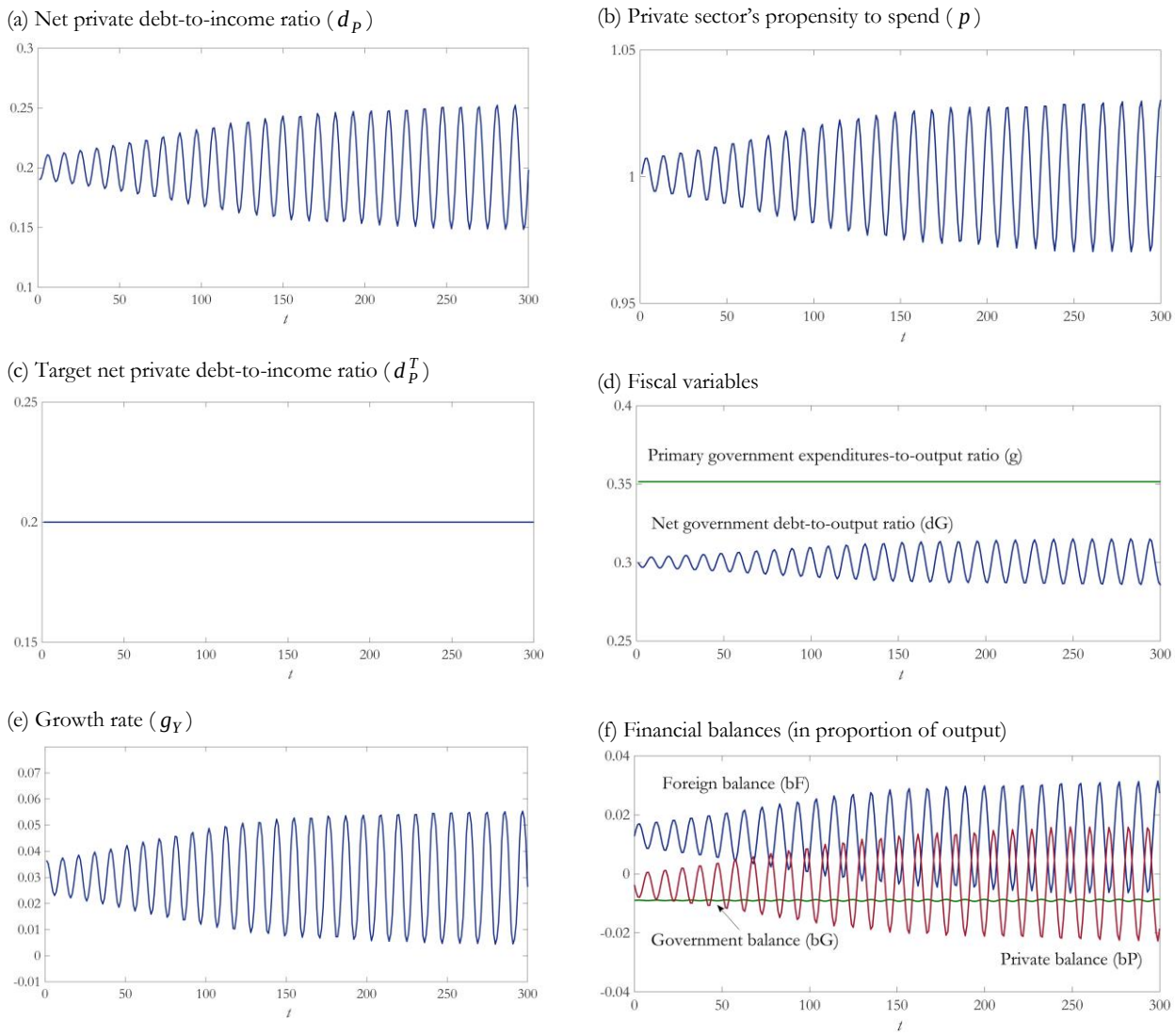


Fig. 2. Limit cycle in the 2D system; private sector in a net debtor benchmark position

$(d_P^B > 0)$; $\lambda = 0.1 > \lambda^* \approx 0.016$

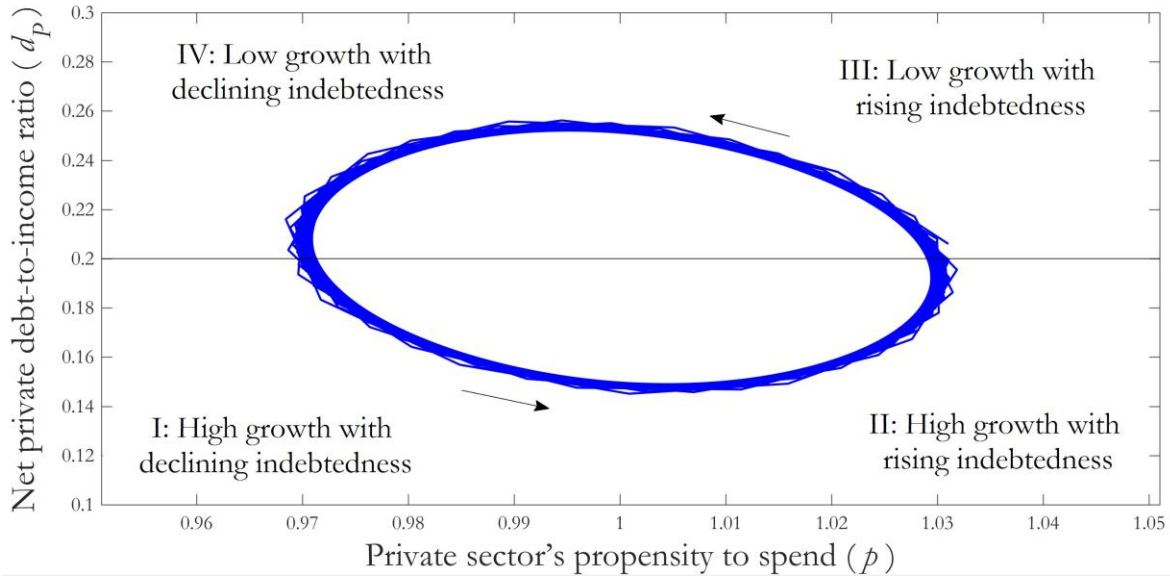
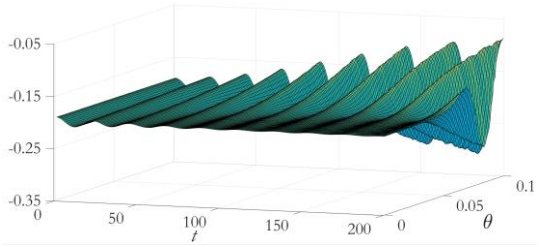
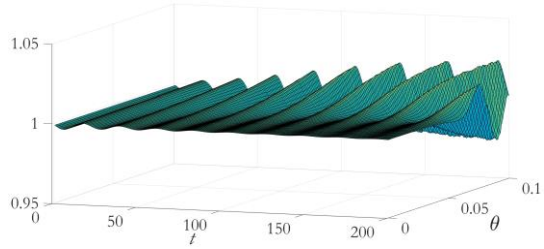


Fig. 3. Dynamic adjustments of the 3D system to a 5% positive shock in the net private debt-to-income ratio for varying values of θ; private sector in a net creditor benchmark position ($d_P^B < 0$).

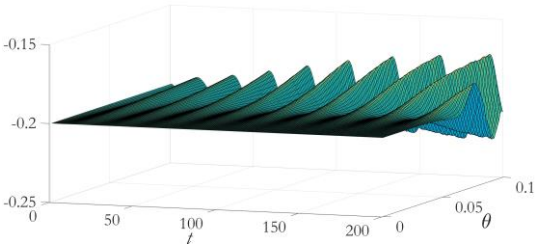
(a) Net private debt-to-income ratio (d_P)



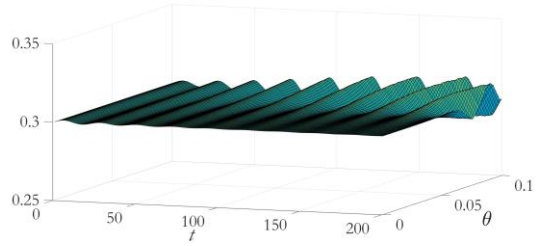
(b) Private sector's propensity to spend (p)



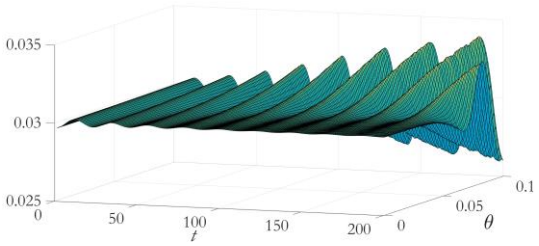
(c) Target net private debt-to-income ratio (d_P^T)



(d) Net government debt-to-output ratio



(e) Growth rate (g_Y)



(f) Government balance-to-output ratio (b_G)

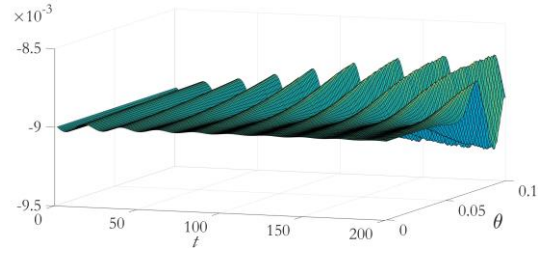
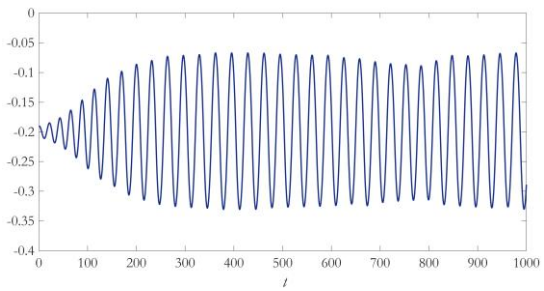
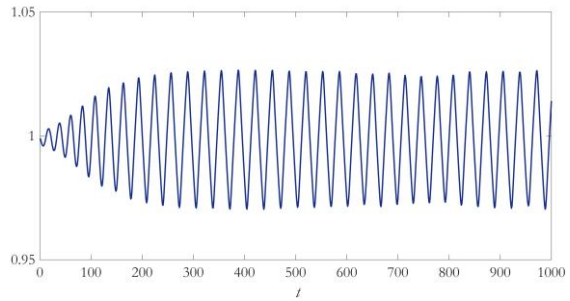


Fig. 4. Dynamic adjustments of the 3D system to a 5% positive shock in the net private debt-to-income ratio; private sector in a net creditor benchmark position ($d_P^B < 0$); $\theta = 0.1 > \theta^* \approx 0.05$.

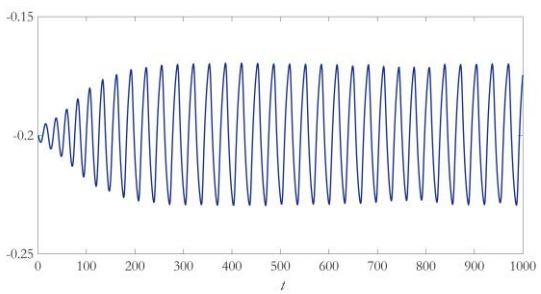
(a) Net private debt-to-income ratio (d_P)



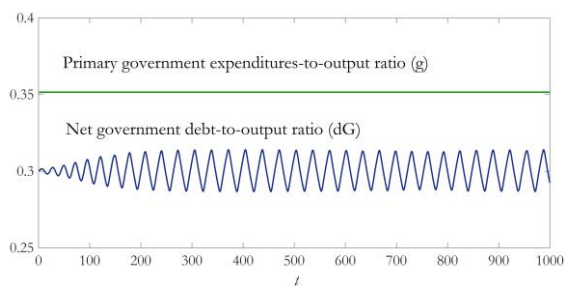
(b) Private sector's propensity to spend (p)



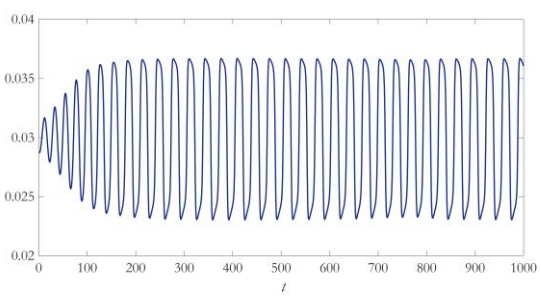
(c) Target net private debt-to-income ratio (d_P^T)



(d) Fiscal variables



(e) Growth rate (g_Y)



(f) Financial balances (in proportion of output)

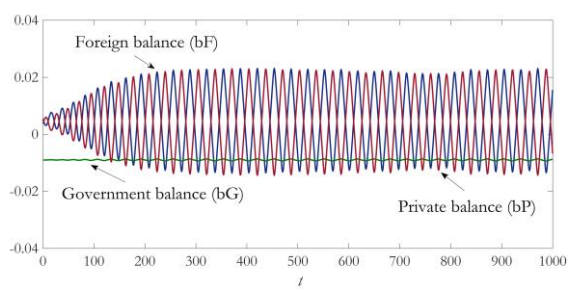
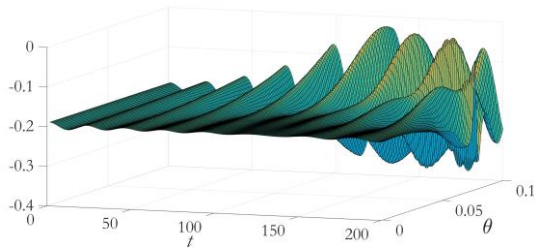
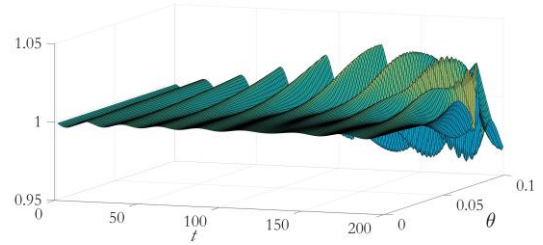


Fig. 5. Dynamic adjustments of the 5D system to a 5% positive shock in the net private debt-to-income ratio for varying values of θ; private sector in a net creditor benchmark position ($d_P^B < 0$); Maastricht-type fiscal rule.

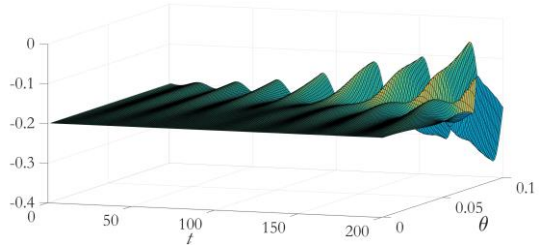
(a) Net private debt-to-income ratio (d_P)



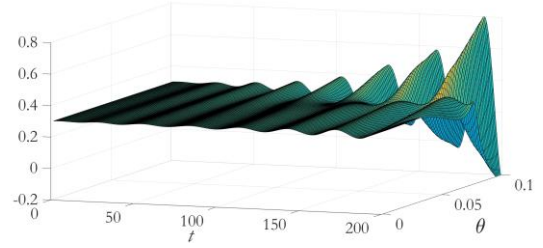
(b) Private sector's propensity to spend (p)



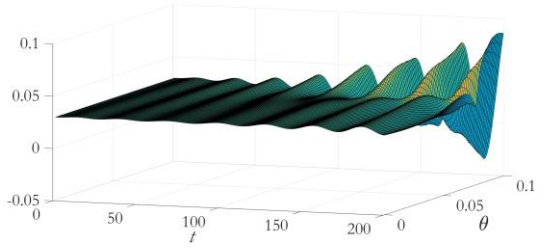
(c) Target net private debt-to-income ratio (d_P^T)



(d) Net government debt-to-output ratio (d_G)



(e) Growth rate (g_Y)



(f) Government balance-to-output ratio (b_G)

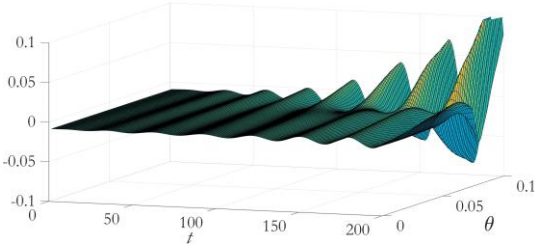
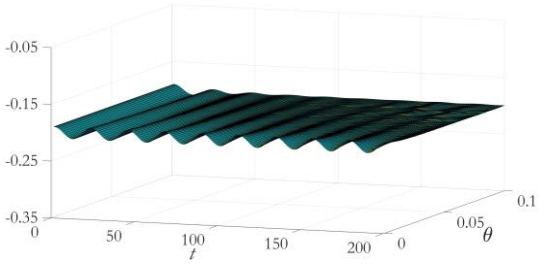
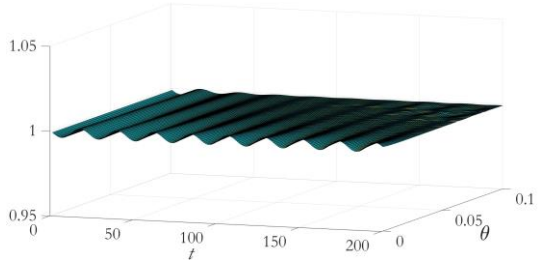


Fig. 6. Dynamic adjustments of the 4D system to a 5% positive shock in the net private debt-to-income ratio for varying values of θ; private sector in a net creditor benchmark position ($d_P^B < 0$); Godley-Minsky fiscal rule.

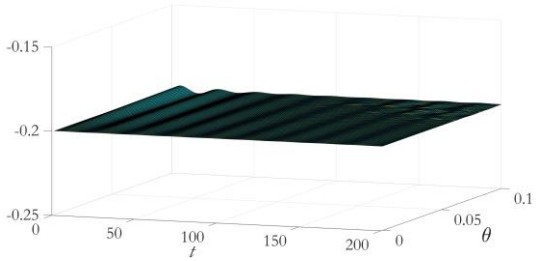
(a) Net private debt-to-income ratio (d_P)



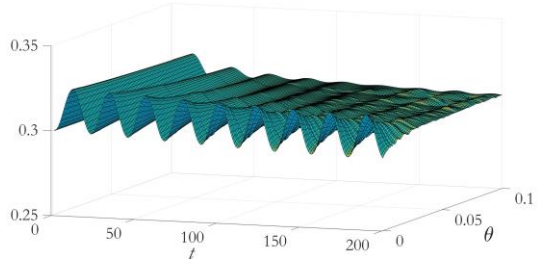
(b) Private sector's propensity to spend (p)



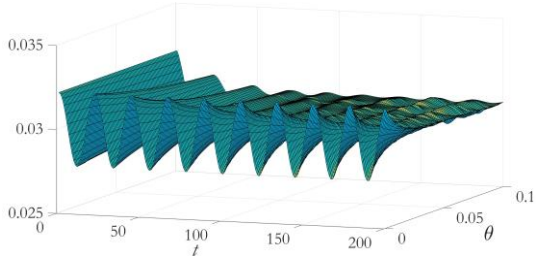
(c) Target net private debt-to-income ratio (d_P^T)



(d) Net government debt-to-output ratio (d_G)



(e) Growth rate (g_Y)



(f) Government balance-to-output ratio (b_G)

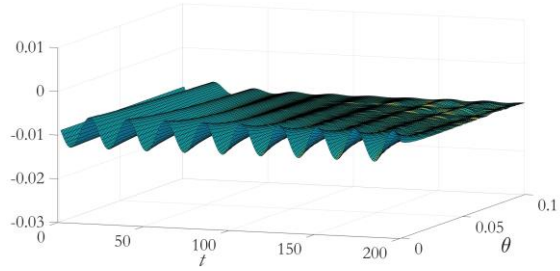
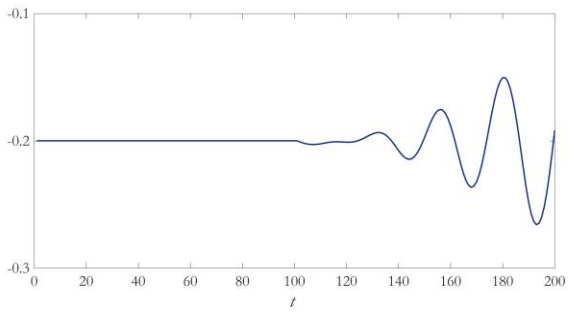


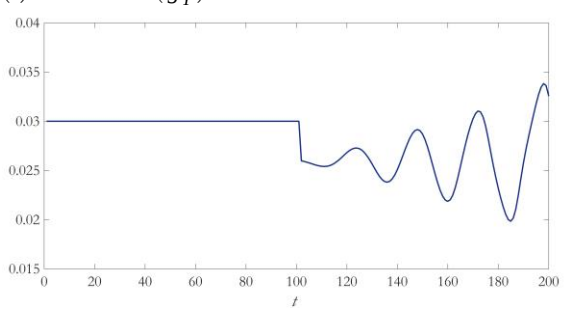
Fig. 7. Dynamic adjustment of the 5D (4D) system to an exogenous reduction in the growth rate of exports (g_X) at $t = 100$ under Maastricht-type fiscal rule (Godley-Minsky fiscal rule); private sector in a net creditor benchmark position ($d_P^B < 0$).

(I) Maastricht-type fiscal rule

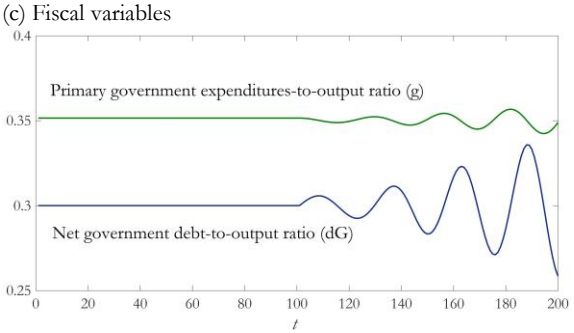
(a) Net private debt-to-income ratio (d_P)



(b) Growth rate (g_Y)

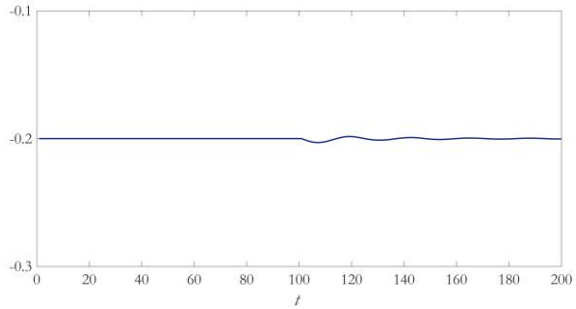


(c) Fiscal variables

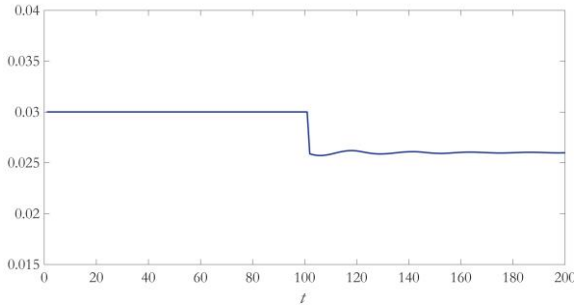


(II) Godley-Minsky fiscal rule

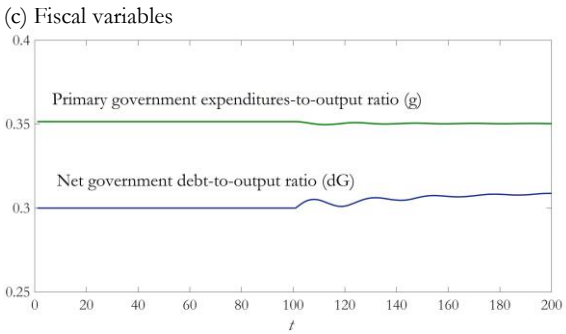
(a) Net private debt-to-income ratio (d_P)



(b) Growth rate (g_Y)



(c) Fiscal variables

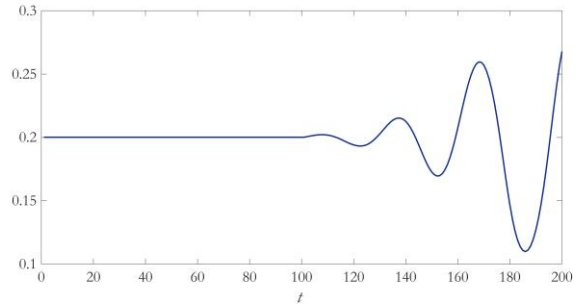


Note: Before $t = 100$*, the system is at its steady state.*

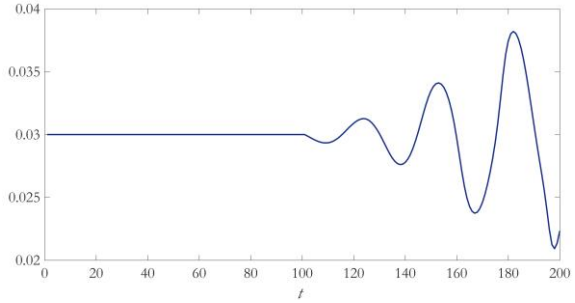
Fig. 8. Dynamic adjustment of the 5D (4D) system to an exogenous increase in the interest rate at $t = 100$ under Maastricht-type fiscal rule (Godley-Minsky fiscal rule); private sector in a net debtor benchmark position ($d_P^B > 0$).

(I) Maastricht-type fiscal rule

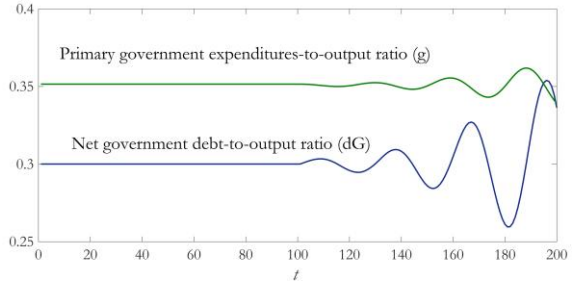
(a) Net private debt-to-income ratio (d_P)



(b) Growth rate (g_Y)

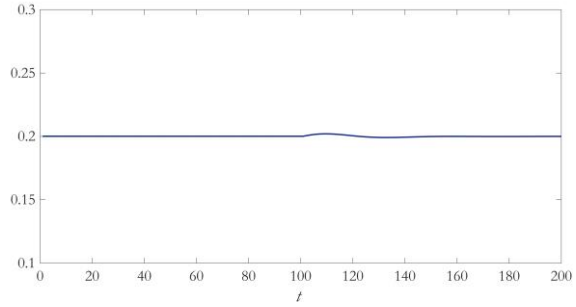


(c) Fiscal variables

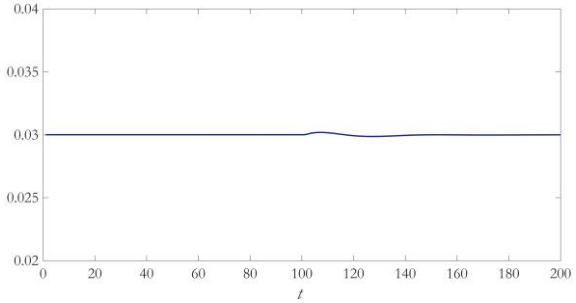


(II) Godley-Minsky fiscal rule

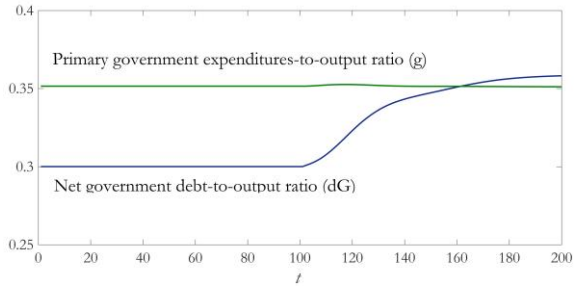
(a) Net private debt-to-income ratio (d_P)



(b) Growth rate (g_Y)



(c) Fiscal variables



Note: Before $t = 100$*, the system is at its steady state. Since* $d_P^B > 0$*, we have* $\lambda < \lambda^*$ *to ensure the stability of the 2D subsystem.*