

Intertemporal Consumption and the Measurement of Inflation: A Dynamic Inertial Price Index

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1. Introduction

The measurement of price inflation has always been a major matter of interest for academics and policy makers. The dangers of using inappropriate indexes to support the design of macroeconomic policies have been forcefully demonstrated by Keynes (1925), who showed that Winston Churchill underestimated the cost of restoring the Gold Standard because he incorrectly used a wholesales index numbers to predict the required price deflation. The US government set up two external independent commissions (the Stigler Commission, 1961, and the Boskin Commission, 1996, endorsed by the Bureau of Labour Statistics - BLS) to assess the accuracy of the measures tracking the change in the level of prices. Their conclusions were not encouraging: quite apart from the technical shortcomings of the standard chained Laspeyres methodology¹, the commission calculated that before 1996 *CPIs* have structurally overstated inflation by 1.3 percentage points on average. The Boskin Commission strongly recommended the BLS to pursue measures of inflation which more accurately capture changes in the cost of living.

The present paper proposes an algorithm for a new index that is able to:

- reduce the structural lagging behaviour with respect to the business cycle of more traditional inflation measures;
- overcome the lack of both a forward- and a backward-looking component in a dynamic approach to the measurement of inflation;
- disentangle the contribution of durable and nondurable goods;
- be easily implementable;
- be more sensitive to the variability of the business cycle.

Thus, through the consideration of both a forward and a backward looking component, the new index provides a better analytical description of consumption dynamics, corrects the inflationary bias of *CPI* and proves more effective in tracking the business cycle. It is therefore a more appropriate indicator to support the design of policy measures.

The rest of this paper is organized as follows. Section 2 provides an overview of the relevant literature, Section 3 derives the new index, Section 4 offers an example of calculation on US data and Section 5 concludes.

2. Statistical and economic price indexes

The academic research classifies the economic price indexes into two main categories:

1. Cost-of-living: where the index is comparing the cost of achieving a fixed standard of living under different price settings;
2. Real consumption: where the index compares the highest standard of living attainable by consumers under two different price settings.

¹ Mainly due to new products, substitution bias and quality change: see Boskin (1998).

The two categories formally result in two alternative analytical approaches, respectively a statistical and an economic one.

The statistical approach adopts a fixed basket principle. A non-random basket of goods is chosen and used to track the change in the prices of its components through time. The price index is defined as the ratio of the value of the representative bundle of goods in two different periods. The best known example of the non-economic approach is the Laspeyres price index on which the calculation of the current *CPI* is based (Laspeyres chained index). Empirical evidence about these indexes points out the need for a different approach, which is closer to the micro and macroeconomic dynamics of consumption (see, among others, Braithwait, 1980, and Nakamura *et al.*, 2015) and, in particular, which is better able to address the adjustment of the consumption bundle over time in response to price shocks.

A new “economic” approach was developed as a response to go beyond the application of the statistical theory: from a Cost-of-Goods (COGS) perspective to a Cost-Of-Living (COLI) one. Formally, an economic index is defined as the result of an optimization programme, typically framed as a cost minimization problem, where the assumptions regarding the cost function play a crucial role in the definition of different price indexes. The traditional static consumption theory supporting the economic approach, though, often resulted naïve and too simplistic to be realistic. During the 1970s, for the first time, the Fisherian intertemporal trade-off in consumption was embedded in a context of price indices by Alchian and Klein (1973). They argued that modern inflation measures were inaccurate, since they neglected asset prices and the allocation of wealth and consumption over time. They addressed the consumption dynamics as a discounted cash-flow problem. Consumers’ choice was however still static, in that they were just considering the discounted value of future prices without a role for expectations and uncertainty. No consideration was given to the optimal consumption path through time dynamically reshaping the optimal consumption bundle.

Shibuya (1992) tried to develop the dynamic framework further. He defined a dynamic measure of inflation called *DEPI* (Dynamic Equilibrium Price Index), which combines an intertemporal optimization process with a no arbitrage condition between the future asset prices and their return measured as the discounted value of future production. Aoki and Kitahara (2010) improved on this by focusing on the role of assets: they assumed Epstein-Zin preferences and considered total wealth rather than durable goods only. Brown and Ying-Lee (2012), by contrast, tried a different approach by turning to the Almost Ideal Demand System framework and proposing a chain price index approach as further step towards a Cost-Of-Living ideal price index. A major contribution came from Reis (2005) with the introduction of a dynamic price index (*DPI*) as the result of a dynamic stochastic general equilibrium model of consumption with asset prices and durable goods. Uncertainty, dynamics and expectations were all contributing to determine the final inflation measure.

We aim to improve the dynamic index proposed by Reis with an emphasis on the inertial drivers of consumption. Building on the seminal work by Brown (1952), we consider that

consumers are mainly concerned about their relative rather than absolute level of consumption and as such lagged variables have an important explanatory role on consumption dynamics. Neglecting them would mean misrepresenting a crucial component of the aggregate demand and therefore undermining the effectiveness of the policy measures relying on it.

3. Intertemporal Consumption and Inflation

The new index we are proposing is deeply rooted in a Cost-Of-Living approach. As analytically shown in this section, it is dynamic, stochastic, and possesses all the classical characteristics of an inflation measure: domain of existence, uniqueness and independence of wealth.

A dynamic price index can formally be defined as a scalar which allows a consumer to maintain the same level of indirect utility after a change in prices. We model habit formation following Muellbauer (1988) and assume the following instantaneous utility function:

$$U(\mathbf{c}_t, \mathbf{c}_{t-1}, \mathbf{s}_t, \mathbf{s}_{t-1}) \quad (1)$$

where $\mathbf{c}_t = (C_{1,t}, C_{2,t}, \dots, C_{N,t})$ and $\mathbf{s}_t = (S_{1,t}, S_{2,t}, \dots, S_{D,t})$ are vectors of non-durable and durable consumption goods respectively. Following Reis (2005), we assume a log-linear functional form modified to account for inertia in consumption:

$$\begin{aligned} &U(\mathbf{c}_t, \mathbf{c}_{t-1}, \mathbf{s}_t, \mathbf{s}_{t-1}) = \\ &= \sum_{j=1}^N \alpha_{1,j} \ln(C_{j,t}) - \gamma \sum_{j=1}^N \alpha_{1,j} \ln(C_{j,t-1}) + \sum_{j=1}^D \alpha_{2,j} \ln(S_{j,t}) - \gamma \sum_{j=1}^D \alpha_{2,j} \ln(S_{j,t-1}) \end{aligned} \quad (2)$$

where $0 \leq \gamma < 1$ is the coefficient of habit persistence and $\alpha_{i,j}$ the share of the consumption bundle allocated to each good (each parameter $\alpha_{i,j}$ lies in the interval $[0,1]$ and all together they add up to 1).

A log-linear functional form is chosen in order to avoid unnecessary analytical complications and because it has two desirable properties: i) the coefficient of relative risk aversion is constant, equal to 1 and coinciding with the elasticity of intertemporal substitution, and ii) there is a parsimonious number of parameters to be estimated.

Durables are both consumption and investment goods, their purchase is episodic and they can be sold before the end of their working life so that their actual cost to the user differs from their purchase cost. To account for this specificity, we assume the contribution of durables to the price index to be their user's cost, defined as follows:

$$u_{j,t+1} = R_{j,t} - \frac{(1-\delta_j)R_{j,t+1}}{I_{t+1}^M} \quad (3)$$

Holding a durable good for one period implies a purchasing price $R_{j,t}$ at date t and then selling what is left over after depreciation δ_j for a price $R_{j,t+1}$ at date $t + 1$. The opportunity cost of investing a sum equal to $R_{j,t+1}$ dollars in durables is $R_{j,t+1}/I_{t+1}^M$ dollars at date t , where $1/I_{t+1}^M$ is the performance yielded by an asset portfolio.

The maximization problem for the consumer can be stated then as follows:

$$V(W_t; \mathbf{p}_t) = \max_{(C_t, S_t)} \left\{ \sum_{j=1}^N \alpha_{1,j} \ln(C_{j,t}) - \gamma \sum_{j=1}^N \alpha_{1,j} \ln(C_{j,t-1}) + \sum_{j=1}^D \alpha_{2,j} \ln(S_{j,t}) - \gamma \sum_{j=1}^D \alpha_{2,j} \ln(S_{j,t-1}) + \beta E_t[V(W_{t+1}; \mathbf{p}_{t+1})] \right\} \quad (4)$$

such that:

$$\sum_{j=1}^N P_{j,t} C_{j,t} + \sum_{j=1}^D R_{j,t} S_{j,t} + \sum_{j \in \{B,E\}} Q_{j,t} B_{j,t} \leq W_t \quad (5)$$

$$W_t = \sum_{j \in \{B,E\}} Q_{j,t} B_{j,t} + \sum_{j=1}^D R_{j,t} (1 - \delta_j) S_{j,t} \quad (6)$$

$$W_t \geq 0 \quad C_{j,t} \geq 0 \quad S_{j,t} \geq 0 \quad (7)$$

The consumer optimally allocates her wealth among durables, $S_{j,t}$, non-durables, $C_{j,t}$, and financial assets, $B_{j,t}$ where the latter can be either equity (E) or bonds (B) and are traded at a vector price $Q_{j,t}$. The maximization problem satisfies the standard budget constraint (5) and the resource constraint (6). The amount of wealth W_t with which the consumer is endowed at the beginning of the period can be increased in two ways: by the payoff $D_{j,t}$ produced by the financial assets or by reselling the durables after depreciation. The third set of constraints (7) consists of non-negativity conditions which rule out Ponzi games and secure a positive level of consumption in every period.

We assume that prices of durables, non-durables and assets are the only sources of uncertainty. They constitute a random vector P_t following, by assumption, a Markov process so that its current realization is a sufficient statistics for expectations. Finally, we assume perfect information by consumers about both the quality of goods and their own tastes, so that they can optimally choose their level of consumption by looking at prices only. The resulting index, being affected by nothing else will be a true price index, *i.e.* defined by the ratio of the expenditure levels needed to attain a reference amount of utility in presence of different prices.

The Dynamic Inertial Price Index can be defined in terms of indirect utility function as the scalar π_t which solves the following difference equation:

$$V(\pi_t W_{t-1}, \mathbf{p}_t, \mathbf{q}_t) = V(W_{t-1}, \mathbf{p}_{t-1}, \mathbf{q}_{t-1}) \quad (8)$$

where \mathbf{p}_t and \mathbf{q}_t , respectively, are a vector of prices for goods and financial assets. Assuming the consumption framework of the previous section and solving for π_t , the *DIPI*

will have the following analytic formulation (for further details about the derivation of the *DIP*I and its components see online Appendix A):

$$\ln(\pi_t) = \frac{(1-\beta)}{(1-\gamma)} \left[T(\ln(P_{j,t-1}), \ln(R_{j,t-1})) - T(\ln(P_{j,t}), \ln(R_{j,t})) \right] \quad (9)$$

where $T(\ln(P_{j,t}), \ln(R_{j,t}))$ is a function of past and current goods prices.

The *DIP*I index is, then, the result of a dynamic stochastic optimization programme whose control vector is the consumption path of mixed durable and non-durable goods. This marks a strong difference with the static index given only by the logarithmic difference of the prices, but it also marks a novelty with respect to the dynamic index proposed by Reis (2005), due to the presence of the inertial component $1/(1-\gamma)$. Furthermore, assuming also that all goods prices dynamics follow a random walk and that the asset returns are i.i.d. we obtain the following closed form formulation of the *DIP*I:

$$\ln(\pi_t) = \frac{(1-\beta)}{(1-\gamma)} \left\{ \sum_{j=1}^N \alpha_{1,j} \ln(P_{j,t}) - \gamma \sum_{j=1}^N \alpha_{1,j} \ln(P_{j,t-1}) + \sum_{j=1}^D \alpha_{2,j} \ln(R_{j,t}) - \gamma \sum_{j=1}^D \alpha_{2,j} \ln(R_{j,t-1}) \right\} \quad (10)$$

A crucial role is played by the coefficient γ measuring the grade of stickiness in consumption, i.e. the smoothing effect: the higher this coefficient the stickier the consumers and, therefore, the weaker the dynamic smoothing effect. Though sharing all the characteristics of a dynamic index, *DIP*I is more general: leveraging on the degree of flexibility of the static response of consumption to an increase in prices, it is able to track a wider range of different behaviours.

When $\gamma = 0$ consumption is not affected by its past level, habit formation plays no role and, therefore, *DIP*I loses its backward looking component. Conversely, when γ is a positive scalar habit formation plays a role in consumers' utility boosting the effect of a change in prices through the coefficient $1/(1-\gamma)$. As $\gamma \rightarrow 1$, *DIP*I would tend to an infinite value, since any difference with the past consumption structure is so great that it prevents consumers from adjusting to the disutility of a change in price.

*DIP*I has important analytical properties, similar to the Dynamic Price Index of Reis (2005) (see online Appendix B for proofs):

Lemma 3.1. Given positive and finite prices and wealth, *DIP*I exists and it is unique.

Lemma 3.2. For any homothetic utility function $u(\cdot)$, *DIP*I is independent of wealth W_t .

Lemma 3.3. *DIP*I is an homogeneous function of degree one.

Lemma 3.4. With homothetic utility function *DIP*I is a superlative index².

² Superlative indexes are, by definition, exact (i.e. equal to the ratio of the values assumed by a function in any pairs of points) for an aggregator with a flexible functional form (i.e. second order approximation for a large class of function in particular for any linearly homogeneous ones).

4. *DIP*: an illustration for the US

We provide an illustration of the (approximate) *DIP* index using US data from 1988 to 2013 to show its feasibility and main characteristics. Raw data for the calculation comes from the Bureau of Labour Statistics (BLS). As in Reis (2005), our sample consists of four non-durables (food, services, apparel and other non-durables) and two durables (shelter and household furnishing) whose price levels and historical weights have been obtained from the official tables of the Bureau.

To calculate the user's cost of durables we relied on the fixed asset tables from the Bureau of Economic Analysis and on the depreciation rates of the corresponding categories of durable goods roughly over the last century. The returns of financial assets are assumed to be i.i.d., they are, then, neutral with respect to the intertemporal optimization process and therefore allow us to focus on consumption itself (see online Appendix C for further details). We assume the dynamic consumption programme (4) – (7) and an inertial coefficient γ equal to 0.27, consistent with estimated values of inertia in consumption (see, among others, Blanciforti and Green, 1983). The coefficients α_1 to α_4 for food, services, apparel and other non-durables are allowed to vary through time and have been obtained by normalizing the raw weights for the *CPI*: Table 1 displays their values.

[Table 1 About Here]

For the durable goods, we considered shelter and household furnishing (Table 1 for the values of α_5 and α_6). Table 2 shows the weights for durables and the time series of their user costs ($u_{5,t+1}$ and $u_{6,t+1}$). These were obtained from the fixed asset table of the Bureau of Economic Analysis (BEA) which provides the depreciation of net stocks and investment for consumers durables.

Figure 1 shows the calculated *DIP* for the US together with the official *CPI* for the period 1988 – 2013. The dynamics of the indexes looks similar for the whole period, but *DIP* clearly shows a higher volatility. In our interpretation this is due to a tighter link with the dynamics of consumption and, therefore, with the business cycle.

[Figure 1 About Here]

General measures of inflation are categorized as lagging indicators of the business cycle (see, among the others King and Watson, 1996). By contrast, in our calculation *DIP* is able to pinpoint the major economic turns anticipating the up/down turns of the *CPI* (e.g. 1997, 2001 and 2007) therefore showing a tighter link to GDP. This is an extremely desirable property as it could reduce the reaction time of counter-cyclical policy measures. To prove a tighter link between our index and the *CPI* we calculated the Pearson correlation index twice. The first time it was computed between the corresponding values of the two series in the same year yielding a value of -0.04: a low and negative correlation. Then we calculated

the Pearson index between the DIPI and the CPI shifted one period ahead: the correlation in this case resulted positive and remarkably higher (0.66).

Comparing the DIPI with the values of the CPI one-period-ahead, the years in which the difference between the two measures is highest are 2001, 2006, and 2008, respectively by about 9, 8 and 11 percentage points. Those are all years in which the economic conditions were particularly stressed³: a further indication of a higher sensitivity of the new index to the underlying economic dynamics⁴.

The years 2000-2008 are particularly instructive for understanding the drivers of *DIPI*. During this period, *DIPI* shows a higher volatility than during the rest of the sample⁵, especially in comparison to the previous 12 years. The reason is twofold: (1) a higher integration (globalization) of the economic systems, which are subject to (2) an increasing number of decentralized shocks. The greater volatility of *DIPI* is very important from a policy point of view: by omitting expectations and dynamic consumption adjustments, *CPI* has to wait for the effects of any shock to work themselves through the economy before it registers a significant change. The greater variability of *DIPI* over the recent period solves the puzzle of the apparent stability of measured inflation, in the face of theoretical predictions of volatile and unanchored inflation during the Great Recession (Miles *et al.*, 2017).

During the whole period *DIPI* has registered an average level of -0.42% as opposed to the 2.67% of the *CPI*. The difference is remarkable in its nature and magnitude. Whilst the standard *CPI* points to an average inflation above the common target of 2%, our index shows a deflationary environment. In such a framework, the former measure would call for a tighter monetary policy, whereas the latter would lead to expansionary monetary and fiscal measures to offset the risk of a deflationary spiral. The average downward cut-off of *DIPI* was of 3.19% with eleven negative values (only one for the *CPI*). The cut-off is in line with the Boskin Commission report (see Boskin Commission, 1996) which argues that fixed basket indexes present a structural upward bias of 1.1 percentage points relative to 'true' inflation. In both cases, the effect is attributed to neglected adjustment of consumption: due to new products for the Boskin Commission, and to the dynamic structure of the consumption decision making for *DIPI*. From a policy point of view, this is extremely significant: if based on *CPI*, policy measures risk being too contractionary, pro-cyclical (with the wrong timing), or both.

Confirming the visual inspection of its dynamics, Table (2) shows the moments of the distribution of the two indexes. *DIPI* displays a lower concentration around the mean (the

³ In 2001, Argentina, the dot-com and the housing bubble, in 2006, again a major housing downturn in the US and in 2008 the global financial crisis.

⁴ It is worth noticing that the relevant shocks are not only financial or economic ones: the path of future consumption, in fact, is also affected by broader political risks.

⁵ The standard deviation is 0.06773 in the window.

standard deviation is 0.043281 compared to 0.01134 for *CPI*) and exhibits longer left tails and negative kurtosis.

[Table 2 About Here]

5. Summary and Conclusions

Building on Reis (2005), our paper sets forth a new dynamic index of inflation accounting for forward- and backward-looking behaviour in consumption.

Our calculations for the US are encouraging and in line with the recommendations of the Boskin Commission. *DIP*I had an average value of -0.42%, identifies a higher number of deflation periods, and was on average lower than the *CPI* by 2.82 percentage points⁶. It shows a higher volatility and the ability of being a leading indicator of the future *CPI* dynamics, thereby being a more useful guide for monetary policy (Miles *et al.*, 2017). The dynamic dimension of *DIP*I increases its elasticity and captures a pattern of consumption adjustment that is completely neglected by the *CPI*.

Important avenues for future research could relate to the role of non-marketed goods (particularly relevant in countries with large shares of public expenditure), consumers' heterogeneity, clustered indexes for specific categories of consumption (e.g. age cohorts) and the harmonization of the different approaches. The derivation of the index from dynamic behaviour, however, and its consistency with economic theory make it an appropriate indicator for guiding monetary policy over the business cycle.

⁶ Regarding the magnitude of the effect it must be noticed, though, that our example was based on a limited (six) number of goods and time range.

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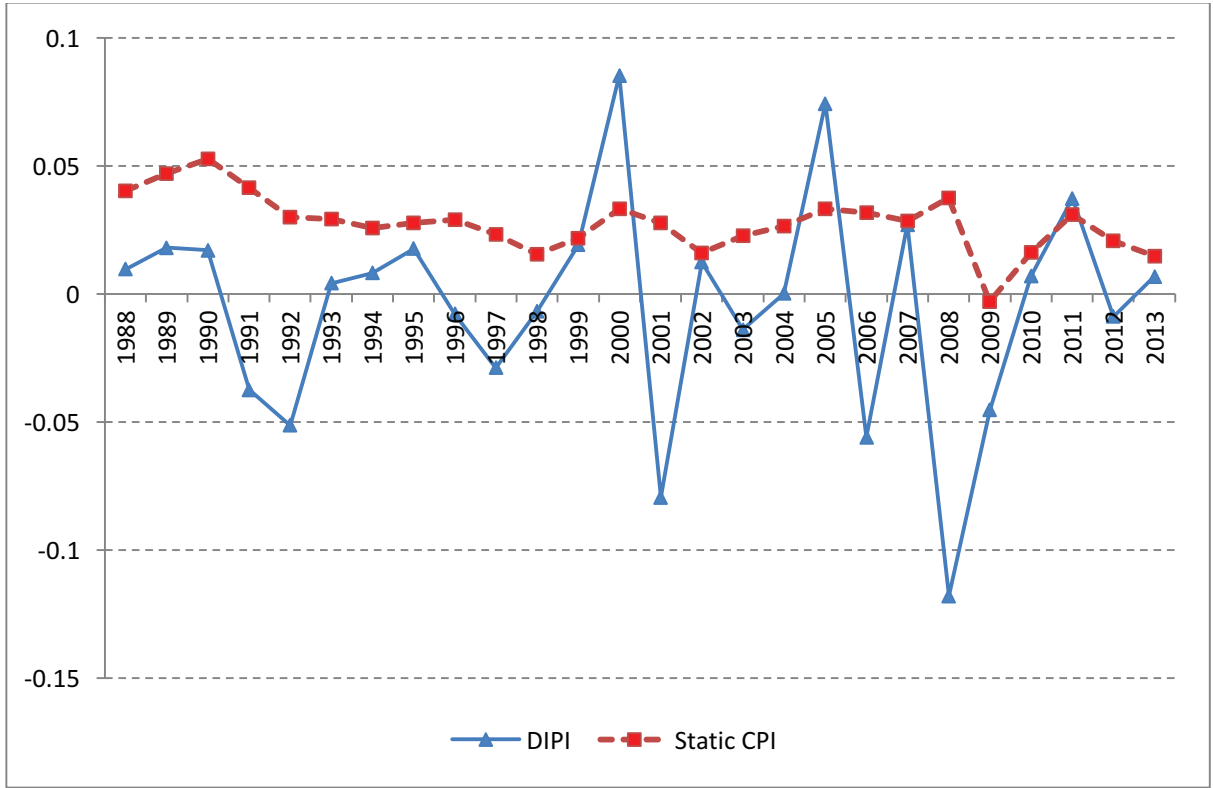
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Figures

Figure 1 – DIPI calculation for US and comparison with the CPI



Tables

Table 1 – DIPI parameters

	NON DURABLES				DURABLES			
	Food	Services	Apparel	Other non-durables	Shelter		Household furnishing	
	α_1	α_2	α_3	α_4	α_5	$u_{5,t+1}$	α_6	$u_{6,t+1}$
1988	0.1308	0.4425	0.0514	0.0946	0.2251	0.1218	0.0556	0.1972
1989	0.1318	0.4434	0.0495	0.0964	0.2253	0.1216	0.0537	0.1983
1990	0.1307	0.4416	0.0490	0.1040	0.2232	0.1235	0.0514	0.1989
1991	0.1289	0.4472	0.0491	0.0992	0.2247	0.1228	0.0509	0.2018
1992	0.1271	0.4505	0.0484	0.0991	0.2246	0.1227	0.0503	0.2011
1993	0.1271	0.4549	0.0475	0.0954	0.2252	0.1215	0.0498	0.2022
1994	0.1277	0.4564	0.0456	0.0955	0.2260	0.1234	0.0488	0.2022
1995	0.1279	0.4592	0.0428	0.0959	0.2266	0.1221	0.0475	0.2046
1996	0.1279	0.4592	0.0428	0.0959	0.2266	0.1233	0.0475	0.2051
1997	0.1256	0.4701	0.0405	0.0882	0.2359	0.1232	0.0396	0.2057
1998	0.1246	0.4679	0.0390	0.0849	0.2447	0.1223	0.0389	0.2062
1999	0.1235	0.4667	0.0378	0.0904	0.2439	0.1222	0.0377	0.2048
2000	0.1224	0.4681	0.0358	0.0932	0.2434	0.1216	0.0371	0.2051
2001	0.1242	0.4732	0.0342	0.0816	0.2502	0.1236	0.0366	0.2051
2002	0.1165	0.4735	0.0338	0.0851	0.2539	0.1223	0.0373	0.2047
2003	0.1141	0.4750	0.0315	0.0832	0.2608	0.1221	0.0355	0.2048
2004	0.1134	0.4740	0.0305	0.0883	0.2593	0.1224	0.0345	0.2042
2005	0.1112	0.4720	0.0302	0.0916	0.2572	0.1209	0.0379	0.2055
2006	0.1099	0.4726	0.0295	0.0916	0.2595	0.1219	0.0368	0.2064
2007	0.1094	0.4644	0.0295	0.1018	0.2578	0.1237	0.0372	0.2070
2008	0.1148	0.4741	0.0290	0.0841	0.2604	0.1232	0.0376	0.2100
2009	0.1083	0.4743	0.0291	0.0978	0.2544	0.1262	0.0362	0.2117
2010	0.1085	0.4738	0.0284	0.1021	0.2524	0.1261	0.0348	0.2109
2011	0.1129	0.4739	0.0281	0.1036	0.2490	0.1228	0.0324	0.2098
2012	0.1127	0.4751	0.0281	0.1028	0.2495	0.1243	0.0318	0.2094
2013	0.1093	0.4803	0.0273	0.0976	0.2519	0.1258	0.0335	0.2098

Table 2 *CPI* and *DIPI*: moments comparison

	Static <i>CPI</i>	<i>DIPI</i>
Mean	0.02762	-0.0042847
Std. Dev.	0.01134	0.0432818
Variance	0.00013	0.0018733
Skewness	-0.23315	-0.4541835
Kurtosis	4.02858	3.827368
Corr static <i>CPI</i>	1.00000	0.0960

Appendices

Appendix A: derivation of the model

The dynamic programming problem of Equations (4) – (7) can be solved by expressing the model in intensive form:

$$\tilde{C}_{j,t} = \frac{P_{j,t}C_{j,t}}{W_t}; \quad \tilde{S}_{j,t} = \frac{R_{j,t}S_{j,t}}{W_t}; \quad \tilde{C}_{j,t-1} = \frac{P_{j,t-1}C_{j,t-1}}{W_t}; \quad \tilde{S}_{j,t-1} = \frac{R_{j,t-1}S_{j,t-1}}{W_t}; \quad \tilde{u}_{j,t+1} = \frac{u_{j,t+1}}{R_{j,t}}$$

The representative consumer's maximization problem is thus:

$$\begin{aligned} V(W_t; \ln(P_{j,t}); \ln(P_{j,t-1}); \ln(R_{j,t}); \ln(R_{j,t-1})) &= \\ &= \max_{\{\tilde{C}_{j,t}, \tilde{S}_{j,t}, B_{j,t}\}} \left\{ \sum_{j=1}^N \alpha_{1,j} \ln\left(\frac{W_t \tilde{C}_{j,t}}{P_{j,t}}\right) - \gamma \sum_{j=1}^N \alpha_{1,j} \ln\left(\frac{W_t \tilde{C}_{j,t-1}}{P_{j,t-1}}\right) \right. \\ &+ \sum_{j=1}^D \alpha_{2,j} \ln\left(\frac{W_t \tilde{S}_{j,t}}{R_{j,t}}\right) - \gamma \sum_{j=1}^D \alpha_{2,j} \ln\left(\frac{W_t \tilde{S}_{j,t-1}}{R_{j,t-1}}\right) \\ &\left. + \beta W_t E_t [V_W(W_{t+1}, P_{j,t+1})] \right\} \end{aligned} \quad (\text{A.1})$$

subject to:

$$W_t = I_{t+1}^M W_t (1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{S}_{j,t} \tilde{u}_{j,t+1}) \quad (\text{A.2})$$

$$I_{t+1}^M = \frac{b_t D_{E,t+1}}{Q_{E,t}} + \frac{(1-b_t) D_{B,t+1}}{Q_{B,t}} \quad (\text{A.3})$$

The dynamic programme becomes:

$$\begin{aligned} V(W_t; \mathbf{p}^t) &= \max_{\{\tilde{C}_{j,t}, \tilde{S}_{j,t}, B_{j,t}\}} \left\{ \sum_{j=1}^N \alpha_j \ln(\tilde{C}_{j,t}) + \sum_{j=1}^D \alpha_j \ln(\tilde{S}_{j,t}) + \right. \\ &- \gamma \sum_{j=1}^N \alpha_j \ln(\tilde{C}_{j,t-1}) - \gamma \ln(W_t) + \ln(W_t) + \\ &- \gamma \sum_{j=1}^D \alpha_j \ln(\tilde{S}_{j,t-1}) - \sum_{j=1}^N \alpha_j \ln(P_{j,t}) + \gamma \sum_{j=1}^N \alpha_j \ln(P_{j,t-1}) + \\ &+ \sum_{j=1}^D \alpha_j \ln(R_{j,t}) - \gamma \sum_{j=1}^D \alpha_j \ln(R_{j,t-1}) + \\ &\left. + \beta E_t [V(W_{t+1}, \ln(P_{j,t+1}), \ln(\tilde{R}_{j,t+1}), \ln(P_{j,t}), \ln(\tilde{R}_{j,t}))] \right\} \end{aligned} \quad (\text{A.4})$$

subject to:

$$W_t = I_{t+1}^M W_t (1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{S}_{j,t} \tilde{u}_{j,t+1}) \quad (\text{A.5})$$

$$I_{t+1}^M = \frac{b_t D_{E,t+1}}{Q_{E,t}} + \frac{(1-b_t) D_{B,t+1}}{Q_{B,t}} \quad (\text{A.6})$$

The envelope condition with respect to the wealth states that:

$$V_W(W_t, \cdot) = \frac{(1-\gamma)}{W_t} + \beta E_t \left[\frac{V_W(W_{t+1}, \cdot) W_{t+1}}{W_t} \right]$$

where $V_W(W_t, \cdot) = \partial V(\cdot) / \partial W$. This implies that:

$$W_t V_W(W_t, \cdot) = \frac{(1-\gamma)}{(1-\beta)}$$

Upon integration, the envelope condition becomes:

$$V_W(W_t, \cdot) = \frac{(1-\gamma)}{(1-\beta)} \ln(W_t) + T[\ln(P_{j,t}), \ln(R_{j,t}), \ln(P_{j,t-1}), \ln(R_{j,t-1})] \quad (\text{A.7})$$

where $T[\cdot]$ is an unknown function.

Dynamic Inertial Price Index

The definition of dynamic price index is:

$$V(\pi_t W_{t-1}, \mathbf{p}^t, \mathbf{q}_t) = V(W_{t-1}, \mathbf{p}^{t-1}, \mathbf{q}_{t-1}) \quad (\text{A.8})$$

It follows that the Dynamic Inertial Price Index (DIPI) will be:

$$\ln(\pi_t) = \frac{(1-\beta)}{(1-\gamma)} \left[T(\ln(P_{j,t-1}), \ln(R_{j,t-1})) - T(\ln(P_{j,t}), \ln(R_{j,t})) \right] \quad (\text{A.9})$$

In the specific case of a logarithmic utility function it can be seen what (A.7) looks like by log-linearizing the maximization problem around its non-stochastic steady state, given the following first order conditions:

$$\frac{\alpha_{j,t}}{\tilde{C}_{j,t}} = \beta W_t E_t [I_{t+1}^M V_W(W_{t+1}, \cdot)] = \beta W_t E_t \left[\frac{W_{t+1}}{W_t (1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{S}_{j,t} \tilde{u}_{j,t+1})} V_W(W_{t+1}, \cdot) \right]$$

and thus:

$$\frac{\alpha_{j,t}}{\tilde{C}_{j,t}} = \frac{\beta(1-\gamma)}{(1-\beta)} E_t \left[\frac{1}{1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{S}_{j,t} \tilde{u}_{j,t+1}} \right]$$

Similarly for durables:

$$\frac{\alpha_{j,t}}{\tilde{S}_{j,t}} = \beta W_t E_t [I_{t+1}^D V_W(W_{t+1}, \cdot)] = \beta W_t E_t \left[\frac{\tilde{u}_{j,t+1} W_{t+1}}{W_t (1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{S}_{j,t} \tilde{u}_{j,t+1})} V_W(W_{t+1}, \cdot) \right]$$

and thus:

$$\frac{\alpha_{j,t}}{\tilde{S}_{j,t}} = \frac{\beta(1-\gamma)}{(1-\beta)} E_t \left[\frac{\tilde{u}_{j,t+1}}{1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{S}_{j,t} \tilde{u}_{j,t+1}} \right]$$

By solving the system above we are able to calculate the non-stochastic steady states of the system as follows:

$$\begin{cases} \frac{\alpha_{j,t}}{\bar{C}_{j,t}} = \frac{\beta(1-\gamma)}{(1-\beta)} E_t \left[\frac{1}{1 - \sum_{i=1}^N \bar{C}_{i,t} - \sum_{i=1}^D \bar{S}_{i,t} \bar{u}_{i,t+1}} \right] \\ \frac{\alpha_{j,t}}{\bar{S}_{j,t}} = \frac{\beta(1-\gamma)}{(1-\beta)} E_t \left[\frac{\bar{u}_{j,t+1}}{1 - \sum_{i=1}^N \bar{C}_{i,t} - \sum_{i=1}^D \bar{S}_{i,t} \bar{u}_{i,t+1}} \right] \end{cases}$$

From the theory we know that $I = 1/\beta$ and from (A.5) we infer that:

$$W = IW \left(1 - \sum_{j=1}^N \bar{C}_{j,t} - \sum_{j=1}^D \bar{S}_{j,t} \bar{u}_{j,t} \right) \Rightarrow 1 - \sum_{j=1}^N \bar{C}_{j,t} - \sum_{j=1}^D \bar{S}_{j,t} \bar{u}_{j,t} = \beta I$$

where the bar-variables indicate the state levels. Therefore the steady states are:

$$\begin{cases} \bar{W} = \beta I \\ \bar{C}_j = \alpha_j \frac{1}{\frac{\beta(1-\gamma)}{(1-\beta)} \frac{1}{\beta}} = \frac{\alpha_j(1-\beta)}{(1-\gamma)} \\ \bar{u}_j \bar{S}_j = \alpha_j \frac{1}{\frac{\beta(1-\gamma)}{(1-\beta)} \frac{1}{\beta}} = \frac{\alpha_j(1-\beta)}{(1-\gamma)} \end{cases}$$

Now from the following relation:

$$f(x, y) = \frac{f_x}{f(x, y)} x x_l |_{ss} + \frac{f_y}{f(x, y)} y y_l |_{ss}$$

where x_l and y_l are log-deviations from the steady states we can evaluate the optimality conditions around their steady states yielding the following log-linearization:

$$\begin{cases} \hat{C}_{j,t} + \frac{(1-\beta)}{\beta(1-\gamma)} E_t \left[\sum_{i=1}^N \alpha_{i,t} \hat{C}_{i,t} + \sum_{i=1}^D \alpha_{i,t} (\hat{S}_{i,t} + \hat{u}_{i,t+1}) \right] = 0 \\ \hat{S}_{j,t} + E_t(\hat{u}_{j,t+1}) + \frac{(1-\beta)}{\beta(1-\gamma)} E_t \left[\sum_{i=1}^N \alpha_{i,t} \hat{C}_{i,t} + \sum_{i=1}^D \alpha_{i,t} (\hat{S}_{i,t} + \hat{u}_{i,t+1}) \right] = 0 \end{cases}$$

From which we obtain the following solution:

$$\hat{C}_{j,t} = 0 \quad \text{and} \quad \hat{S}_{j,t} = -E_t(\hat{u}_{j,t+1}) \quad (\text{A.10})$$

Now, coming back to Equation (A.4):

$$\begin{aligned} & \frac{(1-\gamma)}{(1-\beta)_t} \ln(W_t) + T[\ln(P_{j,t}), \ln(R_{j,t}), \ln(P_{j,t-1}), \ln(R_{j,t-1})] = \\ & = \sum_{j=1}^N \alpha_j \ln(\tilde{C}_{j,t}) + \sum_{j=1}^D \alpha_j \ln(\tilde{S}_{j,t}) + \sum_{j=1}^N \alpha_j \ln(\tilde{C}_{j,t}) + \sum_{j=1}^D \alpha_j \ln(\tilde{S}_{j,t}) + \end{aligned}$$

$$\begin{aligned}
& -\gamma \sum_{j=1}^N \alpha_j \ln(\tilde{C}_{j,t-1}) - \gamma \ln(W_t) + \ln(W_t) - \gamma \sum_{j=1}^D \alpha_j \ln(\tilde{S}_{j,t-1}) - \sum_{j=1}^N \alpha_j \ln(P_{j,t}) \\
& + \gamma \sum_{j=1}^N \alpha_j \ln(P_{j,t-1}) - \sum_{j=1}^D \alpha_j \ln(R_{j,t}) + \gamma \sum_{j=1}^N \alpha_j \ln(R_{j,t-1}) + \\
& + \beta E_t \left[V \left(W_{t+1}, \ln(P_{j,t+1}), \ln(\tilde{R}_{j,t+1}), \ln(P_{j,t}), \ln(\tilde{R}_{j,t}) \right) \right] \tag{A.11}
\end{aligned}$$

By combining the solution (A.10) and the budget constraint (A.5) we obtain the following log-deviation form:

$$\widehat{W}_{t+1} = \widehat{W}_t + \hat{I}_{t+1}^M - \frac{(1-\beta)}{\beta(1-\gamma)} \sum_{j=1}^D \alpha_{j,t} [\hat{u}_{j,t+1} - E_t(\hat{u}_{j,t+1})] \tag{A.12}$$

By replacing (A.12) and (A.10) into (A.11) we have⁷:

$$\begin{aligned}
& T[\ln(P_{j,t}), \ln(R_{j,t}), \ln(P_{j,t-1}), \ln(R_{j,t-1})] = \\
& = \sum_{j=1}^D \alpha_j E_t(\hat{u}_{j,t+1}) - \sum_{j=1}^N \alpha_j \ln(P_{j,t}) + \gamma \sum_{j=1}^N \alpha_j \ln(P_{j,t-1}) + \\
& - \sum_{j=1}^D \alpha_j \ln(R_{j,t}) - \gamma \sum_{j=1}^N \alpha_j \ln(R_{j,t-1}) + \\
& + \beta E_t \left[\hat{I}_{t+1}^M / (1-\beta) + T[\ln(P_{j,t+1}), \ln(R_{j,t+1}), \ln(P_{j,t}), \ln(R_{j,t})] \right]
\end{aligned}$$

Now, assuming a Markov process of order 1 with i.i.d. financial returns $E_t(\hat{I}_{t+1}^M) = 0$ and $E_t(\hat{u}_{j,t+1}) = 0$ and the expression simplifies to:

$$\begin{aligned}
& T_t[\ln(P_{j,t}), \ln(R_{j,t}), \ln(P_{j,t-1}), \ln(R_{j,t-1})] = \\
& = - \left[\frac{\sum_{j=1}^N \alpha_j \ln(P_{j,t}) - \gamma \sum_{j=1}^N \alpha_j \ln(P_{j,t-1}) + \sum_{j=1}^D \alpha_j \ln(R_{j,t}) - \gamma \sum_{j=1}^N \alpha_j \ln(R_{j,t-1})}{(1-\beta)} \right]
\end{aligned}$$

From the DIPI definition:

$$\ln(\pi_t) = \frac{(1-\beta)}{(1-\gamma)} \left[\frac{T_{t-1}(\cdot) - T_t(\cdot)}{(1-\beta)} \right]$$

and, therefore, the DIPI will be:

$$\ln(\pi_t) = \frac{1}{(1-\gamma)} \sum_{j=1}^N \alpha_j \Delta \ln(P_{j,t}) - \gamma \sum_{j=1}^N \alpha_j \Delta \ln(P_{j,t-1}) + \sum_{j=1}^D \alpha_j \Delta \ln(R_{j,t}) - \gamma \sum_{j=1}^N \alpha_j \Delta \ln(R_{j,t-1})$$

⁷ From the analytical derivation a term $\gamma\beta \ln(W_t)$ remains as the only link with wealth. For non-abnormal values of wealth, it can be neglected given its small magnitude after the multiplication and the error is smaller the smaller are γ and β .

*Appendix B: proofs of lemmas***Proof of Lemma 3.1.**

A consumption problem has a solution as long as the value function is monotonically increasing in its arguments. From the DIPI definition (9) as π goes from 0 to $+\infty$, the left hand side of the equation increases monotonically from $-\infty$ to $+\infty$. Given positive wealth and prices, the right hand side is a finite number and therefore the solution to the overall problem exists and it is unique.

Proof of Lemma 3.2.

Following Carroll (2000) we have modelled inertia in consumption as a ‘subtractive habit’. This does not preclude the possibility of having homothetic preferences over consumption⁸. With a plain algebraic manipulation of Equation (8), then, this lemma is proven.

Proof of Lemma 3.3.

As in Reis (2005) the proof comes from the index definition. The following transformation can be applied to Equation (8): $\{\{W_t, p_t, D_t\} \rightarrow \{aW_t, ap_t, MD_t\}\pi_t\}$. The transformation doesn’t affect the feasibility set of the maximization problem nor does the objective function and the value function.

. With homothetic utility function the DIPI is a superlative index.

Proof of Lemma 3.4.

Diewert (1976) defines as Superlative a Cost-Of-Living (COLI) index which can be calculated for a specific flexible functional. Referring to the indirect approach to consumer’s choices it is, then, the change in the expenditure function due to a change in relative prices. Superlative indexes are, by definition, exact (i.e. equal to the ratio of the values assumed by a function in any pairs of points) for an aggregator with a flexible functional form (i.e. second order approximation for a large class of function in particular for any linearly homogeneous ones). If a homothetic utility function is assumed the index number resulting from its optimization would be also locally optimizing any of its (second order) approximations. The resulting DIPI index will, then, be superlative too.

⁸ See Gomis-Porqueras and Bossi (2009).

Appendix C: DIPI calculation for US

Given the i.i.d. assumption on financial returns our starting point is the following simplified log-linearized definition of the DIPI where differences of order higher than one are neglected:

$$\begin{aligned} \frac{\ln \pi_t}{(1-\beta)} = \frac{1}{(1-\gamma)} & \left\{ \sum_{i=0}^{\infty} \beta^i \left(\sum_{j \in N} \alpha_j (E_t[p_{j,t+i}] - E_{t-1}[p_{j,t-i+1}]) \right. \right. \\ & + \sum_{j \in D} \alpha_j (E_t[r_{j,t+i}] - E_{t-1}[r_{j,t-i+1}]) \\ & - \sum_{i=0}^{\infty} \beta^i \sum_{j \in D} \alpha_i \chi_i (E_t[\Delta r_{j,t+i} - i_{t+i}^M] - E_{t-1}[\Delta r_{j,t+i} - i_{t+i}^M]) \\ & \left. \left. - \sum_{i=1}^{\infty} \beta^i \sum_{k=1}^i (E_t[i_{i+k}^M] - E_{t-1}[i_{t-1+k}^M]) \right) \right\} \end{aligned}$$

Now, with some algebraic manipulations, we obtain, at the end of this section, a friendly expression of the above expression which is the base of our calculation.

The stochastic processes assumed for prices imply the following level VAR representation:

$$p_t - \mu = \Phi_1(p_{t-1} - \mu) + \dots + \Phi_k(p_{t-k} - \mu) + \varepsilon_t$$

Following Hamilton (1994) this can be rewritten as a 1st order VAR on the vector of prices: $p^t = [p_t - \mu, \dots, p_{t-k+1} - \mu]^T \cdot F p^{t-1} + v_t$ where F is a $np \times np$ matrix of prices. Therefore it will be:

$$E_t[p_{t+s} - \mu] = F_{11}^{(s)}(p_t - \mu) + \dots + F_{1k}^{(s)}(p_{t-k+1} - \mu) = F_1^{(s)} p^t$$

Defining the expression $A = \sum_{i=1}^6 \alpha_i e_i$ where e_i is a row vector 1×6 with 1 in the i^{th} column and zero in the others it will be also:

$$\sum_{i=0}^{\infty} \beta^i A (E_t(p_{t+i}) - E_{t-1}(p_{t-1+i})) = \sum_{i=0}^{\infty} \beta^i F_1^{(i)} \Delta p^t$$

with respect to durables and we define $B = \alpha_5 \chi_5 e_5 + \alpha_6 \chi_6 e_6 - (\alpha_5 \chi_5 + \alpha_6 \chi_6) * R$ where R is the composite return of the financial assets which we assumed to be 0.05 and where $\chi_j = (1 - \bar{u}_j) / \bar{u}_j$ with \bar{u}_j the steady state level of the user cost. It will be, then:

$$\frac{\ln \pi_t}{(1-\beta)} = \frac{1}{(1-\gamma)} \sum_{i=0}^{\infty} \beta^i [(A + B)F_1^{(i)} - BF_1^{(i+1)}] \Delta p^t$$

Which is the basis of our calculation.

From an analytical point of view what makes the former expression different from the exact DIPI we derived in the paper (and therefore an approximation of its exact value) is the following:

- we adopted dynamic weights for the goods. In the text we assumed fixed α s whereas in the calculation we tracked their levels on a year-by-year basis according to their revisions by BLS. This was aimed to have a fairer comparison with the CPI;
- we neglected price differences higher than 1. This was aimed to simplify the analytical structure of the calculation.

Given the small number of goods in the sample and the other assumptions regarding financial returns and the coefficient of habit persistence the overall effect of the former choices on the final result was expected to be negligible.