

Dynamic, small-world social network generation through local agent interactions

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October 7, 2013

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Abstract

To model agent relationships in agent-based models, it is often necessary to incorporate a social network whose topology is commonly assumed to be “small-world”. This is potentially problematic, as the classification is broad and covers a wide-range of network statistics. Furthermore, real networks are often dynamic, in that edges and nodes can appear or disappear, and spatial, in that connections are influenced by an agent’s position within a particular social space. These properties are difficult to achieve in current network formation tools. We have therefore developed a novel social network formation model, that creates and dynamically adjusts small-world networks using local spatial interactions, whilst maintaining tunable global network statistics from across the broad space of possible small-world networks. It is therefore a useful tool for multi-agent simulations and diffusion processes, particularly those in which agents and edges die or are constrained in their movement within some social space. We also show, using a simple epidemiological diffusion model, that a range of networks can all satisfy the small-world criterion, but behave quite differently. This demonstrates that it is problematic to generalise results across the whole space of small-world networks.

1 Introduction

Agent-based models are a useful tool within the social sciences for analysing emergent behaviour from human systems by capturing the complex, low-level interactions of the

participating agents [1]. Researchers and policy makers have begun to use such an approach to study real world problems [2], with many of the models focusing on contagion dynamics within a given system, such as the spread of disease [3], foreclosure [4], mental states [5] and social norms [6].

Such contagion dynamics rely on embedding a social network within the model, representing a dependency connecting individuals in the system, such as friendship or beliefs. However, whereas much of network theory is based around static networks [7], agent-based systems modelling human behaviour require dynamic networks that can both form and evolve by virtue of local interactions between the agents, whilst maintaining certain characteristics for the network as a whole [2], such as key statistics gathered from empirical evidence. Consequently the topology of the social network within these models can dramatically affect the system, with shifts in agent relationships forming an important part of the dynamics of system behaviour [8, 9]. Furthermore, authors such as Roth [10] have questioned the true empirical value to social science of models that utilise dubious low-level behavioural assumptions to drive network interactions, so it is important to have the dynamics governing network topology influenced by some realistic spatial relationship between agents. One example of this is in the study of human migration [11], where the likelihood of an individual migrating is dependent on both the individual's material circumstances, but also their *perception* of migration. This is altered by the migration decision of the individual's friends and family as specified by their social network, a dynamic entity whose evolution is partly affected by the geographical proximity of individuals.

As well as being static rather than dynamic, another issue with incorporating social networks into agent-based models is that they are partitioned into very broad classes. For example, in a recent study on migration [11], the simulation tool used could generate networks that were either regular, random or small-world. While the first two classes are well-understood, the concept of what a small-world network is, beyond the familiar

concept of “six degrees of separation” postulated by Stanley Milgram [12], is more opaque. Indeed, though there exist methods to precisely define whether a network has the small-world property [13], this is nonetheless a broad classification and it is unclear whether such a definition is granular enough to capture the key characteristics that determine system properties; that is, whether all networks which are deemed small-world will have the same effect when used in a multi-agent model, a concern echoed in the work of Thiriot [14]. As being small-world is a common feature of seemingly all real-world social networks [15], such a network topology could be an important determinant in real-world simulations.

Agent-based systems therefore require a formation model that can create a dynamic network, where topology is determined by spatial interactions, but that can also create networks with consistent characteristics across the spectrum of possible small-world networks. Two such characteristics are the clustering coefficient (the degree to which agents in a network create tight knit groups) and the mean shortest path (the average shortest distance between any two agents in the network). We therefore propose a new network formation model, which creates and maintains links through localised pair-wise interactions caused by agent collisions in continuous time, and is able to dynamically satisfy given global constraints for clustering, mean shortest path and network density which classify it as small-world.

By altering several key parameters of the model which affect the movement of the agents, we are able to generate a large set of contrasting networks, which all fit within the small-world space, but display vastly different characteristics. We therefore firstly demonstrate the risks associated with current small-world measures and show that generalising results across the small-world space is incorrect. Secondly, we analyse the tension between dynamism and stability within these networks.

2 Related literature

Perhaps the most well-known model for creating small-world networks is that developed by Watts and Strogatz (“WS”) [16]. This model takes a regular lattice network and “rewires” a proportion of the links to create several weak, long-range ties, causing the network to maintain the high clustering coefficient inherent in a lattice, whilst substantially reducing the mean shortest path. Although excellent and comprehensive for creating a static, small-world network, it cannot be extended to a dynamic network whilst maintaining the same characteristics, or grow networks from scratch.

There are however many models that can grow and scale networks in a stable manner. Barabási and Albert [15] introduce the concept of preferential attachment, whereby new nodes added to a network are more likely to attach themselves to high-degree existing nodes. This is postulated as an explanation for the “scale-free” behaviour they have encountered in some social networks, where the degree distributions seem better fitted by a power law than a standard Gaussian. Jackson and Rogers [17] emphasise the need for both random meetings and network-based meetings when new nodes attach to a network, and analyse how changing the balance of the two can lead to different degree distributions. They then demonstrate that the resulting networks possess key social network characteristics. Handcock and Morris [18] introduce an elegant Markov Chain Monte Carlo algorithm, which allows them to create networks satisfying a given degree distribution that probabilistically cover the full range of possible clustering coefficients. However this requires applying a degree distribution a priori and is not able to maintain a dynamic network with its properties in equilibrium.

All of these models provide networks that grow and scale in a stable manner, but do not have the capacity to maintain stability when connections are *altered* instead of added. Indeed, disrupting something as simple as the constant growth rate in the Barabási-Albert model can have a major influence on the degree distribution [10]. Therefore we require

a different approach in order to *maintain* stability in dynamic networks.

The first researchers to examine how stable, dynamic, small-world networks can emerge from localised interactions were Davidsen, Ebel and Bornholdt [19], who demonstrate that “transitive” linking (i.e. friends of friends being introduced) and a finite life for each node can lead to a stable network, which exhibits clustering and path length statistics that are small-world in nature. The network evolves slowly by having a turnover rate for nodes and edges. It does require an existing network before evolution can take place however. The closest model to our own and the inspiration for our approach is González *et al* [20], who place a set of particle nodes in a periodically bounded square cell and assign them a random direction and initial velocity. Particles that collide will form a link in the network, but will also increase in speed, leading to a preferential attachment-style model with the network formed displaying a scale-free distribution. Stability is obtained by nodes dying in a similar manner to Davidsen *et al* [19], but the randomness of collisions mean that there is no concept of transitive linking or network-based meetings.

A related strand of research highlights the potential dangers inherent in using unsuitable network formation models. Roth [10] focuses on the plausibility of the interaction behaviours between individuals and advocates the use of empirical testing before any modelling work is undertaken. Similarly, Thiriot [14] highlights the dangers of generalising the results from a single formation model to apply to a broad class of networks: for example, generalising a result on networks formed by the WS model to the space of all small-world networks. By adopting an agent-based approach that allows the stochastic development of links based on both random meetings and existing acquaintances, we believe that our model is flexible enough to create dynamic networks across the small-world space by using plausible interactions.

The idea of “cross-pollination” between the fields of social networks and agent-based systems is growing as each finds that it requires tools from the other, such as the need for simulations to model more complex relationships in social network theory. Carley

[21] has introduced the field of Dynamic Network Analysis, which combines multi-agent modelling with learning and a “meta-matrix” approach to network representation. This is used to tackle dynamic, high-dimensional, real-world networks such as those of terrorist cells. Grant [22] uses an agent-based model to investigate how embedded hierarchical relationships affect the chain of command in military reporting, while Savarimuthu [6] utilises the collision-based network formation model of González [20] to model the spread of social norms. However these papers only utilise networks covering a small part of the space of possible small-world networks and so are limited in their conclusions.

3 Methods

3.1 Overview

The fundamental idea behind our model is that a network is organically created by collisions between agents moving through a “social space” [20], representing any dimension where social interactions take place (examples could be real-world person-to-person meetings, or on-line meetings through social media). The topology of the social network can be controlled by altering parameters of the agent’s movement, such as the distance it can travel. We will now discuss in detail the necessary mechanisms for this to occur.

3.2 Network formation

The environment for the network formation will be a 2-dimensional torus representing some “social space”, a dimension in which the Euclidean distance between any two objects will represent a social distance. A fixed number of agent particles are distributed randomly on the torus, with each agent representing a potential node in the social network. The starting point for each agent will be defined initially as the home position and represents the point to which the agent will return after a journey. Each agent will

be instantiated with a random angle of travel and a random range, which is drawn from a negative exponential distribution defined below. Together these two variables define a fixed target point Θ for the agent to travel to.

To calculate the range for an agent s , a random number $x \in [0, 1]$ is mapped onto the negative exponential distribution:

$$agent\ range = -\ln\left(\frac{x}{\lambda(1 + \mu w_s)}\right) \quad (1)$$

where w_s is the number of direct contacts or “neighbours” of s , and λ and μ are fixed parameters of the system. As the angle of travel is drawn randomly from $[\pi, \pi]$, the locus of travel for the agent is technically infinite, even if large distances are unlikely. As long as $\mu > 0$, the average distance of travel will decrease as the number of neighbours increases.

The range of movement for the agent can be thought of as equivalent to a dog on a lead of fixed length, tied to a post. From a social network viewpoint, we can view this as each agent occupying a fixed base within the social space, from which it can make forays to meet others. The crucial factor here is that the probability of forming a connection with someone nearby in the social space is much greater than for someone further away. This concept allows both random and network-based meetings.

Movement of the agent is categorised into three separate stages:

Outward movement: after establishing a target away from its base position, the agent moves at a constant velocity along the vector between its current position and its target. Once the target has been reached, the agent will stop and reassign its target to be its original home position.

Homeward movement: the agent moves at a constant velocity along the vector between its current position and home position. Once home is reached, the agent will randomly redraw an angle and range from the distributions above to establish a new

target.

Post-collision movement: after colliding with another agent, the agent will adjust its home position according to the specified collision rules below. It will then randomly redraw an angle and range from the distributions above and establish a new target, regardless of whether it was previously on its outward or homeward movement. This allows an agent to potentially travel further from its home and explore a new area of social space.

If two agents pass within a radius width of each other on the torus, they are defined to have collided. This causes an undirected edge to be formed between the two agent nodes, representing some social relationship

After the collision of agents r and s , agent r will redefine its base position h_r according to the following formula:

$$h_r^{new} = h_r + \mathbf{v}_{rs} \left(\frac{\alpha}{\beta w_r + 1} \right) \quad (2)$$

where w_r is the number of neighbours of r and \mathbf{v}_{rs} is the vector from h_r to h_s . Agent s will update its home position in a similar manner. This effectively causes the homes of the agents to be drawn towards each other, with the magnitude of the “gravitational pull” dependent on the parameters α and β , as well as the number of neighbours. α can be thought of as a “global” gravity, and β as a “local” gravity.

3.3 Network stability

If λ and μ are set so that the expected locus of an agent s is always greater than the size of the torus, then eventually every agent will collide with every other agent to form a complete network. This is undesirable from an analysis point of view, as we need our networks to be sparse [23]. It is therefore important to utilise the idea of Davidsen *et al* [19] and implement some form of cost function associated with edge formation. One

method of achieving this is by increasing the value of μ , which will limit the distance an agent with many connections is able to move within the social space. However if we do not wish to restrict movement to this degree, we can impose some age limit on either the connections or the agents themselves.

González [20] implements the latter by drawing the maximum age of each agent node uniformly from a fixed range, increasing the node’s age with each time-step, and finally killing the node off once it reaches the maximum. The agent node is “reborn” at the same point in the social space, but without any of its connections. This is sufficient to create stability in that model as each node has no fixed home within the social space. However, unless the home position of the node is changed in our model, it will quickly re-establish its old connections and the mass of nodes will slowly coalesce to form an extremely dense super-cluster. Therefore, on rebirth, the base position of the agent node will be randomly reset to anywhere within the torus.

While the node ageing method can be justified as agents leaving and joining a social network, it does not allow for the concept of single link deletion, which is very common in most network structures. Therefore we must also consider an edge ageing based model, where edges age in the same way as nodes do above. From a stability viewpoint, this method is more problematic due to the fact that we cannot reset a node’s home position whenever an edge dies, as other edges continue to exist. Therefore we will have to implement a shock to the agent, which will be equivalent to a collision with another node at a random point in the social space. This shock will be inversely proportional to the number of neighbours that the node has, so a well-connected agent will stay within the same region of the social space.

3.4 Network statistics

In order to distinguish networks within the space of all small-world networks, we need a measure to capture “small-worldness”. The *S score* of a network Z , developed by

Humphries and Gurney [13], is defined as:

$$S_Z = \frac{\Gamma}{\chi} \quad \text{where } \Gamma = \frac{C_Z}{C_{rand}} \quad \text{and } \chi = \frac{SP_Z}{SP_{rand}} \quad (3)$$

C_Z and SP_Z are the global clustering coefficient and mean shortest path of the network, while C_{rand} and SP_{rand} are the global clustering coefficient and mean shortest path for an equivalent random network with the same number of nodes and edges, created using the Erdős-Rényi method. The idea is simple but effective, in that it combines the three key measures of clustering, mean shortest path length and density into a single number. An *S score* of 1 or below indicates no small-worldness, whereas an extremely high *S score* is possible in extreme networks of low density and high clustering.

4 Results

4.1 Key parameters

We demonstrate how five key parameters of the model can affect the evolved network:

- λ , which controls agent range in the negative exponential distribution.
- μ , which reduces agent range in proportion to the number of neighbours in the negative exponential distribution.
- α , which implements an attractive force between the bases of newly collided agents, acting as a “global gravity”.
- β , which reduces the attractive force between the bases of newly collided agents in proportion to the number of neighbours.
- M , which is the maximum age for either a node or an edge in the network.

We demonstrate how different parameter combinations can produce networks with a variety of statistics across the small-world space, as well as how they affect the dynamism of the network. All scenarios will be generated using one thousand agents.

4.2 Network stability

An important result which will influence the other statistics is if and when a giant cluster is formed in the network, meaning that any node can be reached by any other node. González [20] is able to extend results from two dimensional percolation theory to find a critical exponent for the number of collisions that an agent has during its lifetime. At this critical exponent, the number of separate clusters in the network drops rapidly, representing a percolation transition. Our model differs in that particles do not have complete freedom to move throughout the torus, so we cannot use the same theory. Instead we can investigate whether a transition still occurs, dependent on the movement and age of the agents.

The first key parameter is λ which controls the range of agent movement. By decreasing λ and holding the other variables steady, we find that the network moves from a set of disparate clusters to one giant cluster with a sharp transition around $\lambda=0.2$. If μ is increased, the transition takes place at a slightly lower value of λ due to the reduced available movement for the nodes.

The other key parameter for the formation of the giant cluster is the maximum age M , either of the nodes or the edges. When the edges are aged instead of the nodes, the transition point to the giant cluster is much lower, around $M=200$ versus $M=500$, due to the fact that an edge dying will not necessarily remove a node from a giant cluster in the same way that a node dying will.

Similarly, long term stability was also investigated. Networks are able to maintain consistent global statistics up to at least 300,000 timesteps unless $\alpha>2$, in which case the nodes coalesce into a “super-cluster”, forming a complete network.

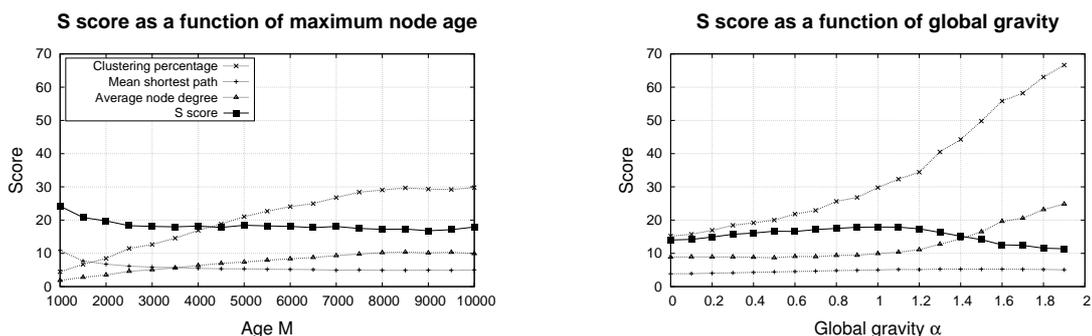
Therefore, in order to enforce a stable giant cluster, all networks will be generated with a maximum $\lambda=0.1$, maximum $\alpha=2$ and minimum $M=2000$.

4.3 Parameter investigation

We run a series of simulations for one thousand nodes under a variety of different parameter combinations to understand how each influences the topology of the network formed. All simulations are run for $2M$ timesteps (where M is the maximum node or edge age), which empirical testing revealed to be sufficient for the global statistics of the network to settle into a stable state.

Each parameter is investigated in turn, with the remaining parameters held at a “default” level decided by empirical testing. Although this does not capture correlation between parameters, it is sufficient to determine how each one broadly affects the key statistics of clustering coefficient, mean shortest path and network density. Any instance where parameter correlation has a specific effect on the network will be discussed independently.

FIG. 1. Dependence of network statistics on parameters M and α . The default network has parameters $\lambda=0.005$, $\mu=1$, $\alpha=1$, $\beta=1$ and $M=10,000$, with M and α amended in turn.



Increasing either M or α leads to an increase in clustering and density, with little change in the mean shortest path (Figure 1). For the former, this is simply because a longer average life for each agent leads to more collisions and a higher average degree (Figure 1, left panel). For the latter, this is due to the fact that for any two nodes

that form a new edge, their homes within the social space will move closer together (Figure 1, right panel). When α is small ($\alpha < 1$), the “gravity” between the homes is only strong enough to pull them slightly closer together, with this effect dampened further if the number of existing links that a node has is high. This causes a small increase in network-based meetings for newly-linked agents, as they are now more likely to move through the social space of the other, leading to an increase in network clustering but no corresponding increase in density. However as α grows larger and the gravitational effect becomes stronger, more and more of agents will be sharing the same social space for their movement, leading to a modular, highly-clustered network. Eventually, if α is made too large, all agents will coalesce into an unstable super-cluster which will form a complete network. Therefore M must be correspondingly reduced as α is increased in order to maintain stability.

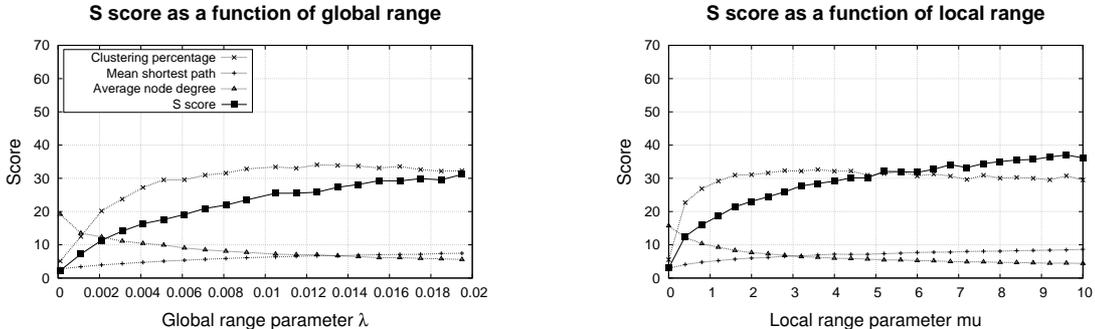
However it should be noted that α cannot influence the network in isolation, as its effect is dependent on the value of β . A special case is when $\beta=0$. By analysing the formula for updating the agent home, we see that:

$$h_r^{new} = h_r + \mathbf{v}_{rs} \left(\frac{\alpha}{\beta w_r + 1} \right) = h_r + \mathbf{v}_{rs} \alpha \text{ for } \beta = 0 \quad (4)$$

As the global gravity now has no dependence on the number of neighbours each agent has in the network, setting $\alpha=0.5$ will mean that on collision, the homes of both agents involved will converge to the same point. This causes a large increase in clustering as agents instantly “share” their social space with any new contact, but does not affect the network density as agents are still able to move large distances in the social space, thus precluding the formation of super-clusters. Therefore this is a very effective method of increasing clustering without disrupting the other network statistics.

Apart from its relationship with α , β provides few other interesting dynamics and will not be studied in detail.

FIG. 2. Dependence of network statistics on parameters λ and μ . The default network has parameters $\lambda=0.005$, $\mu=1$, $\alpha=1$, $\beta=1$ and $M=10,000$, with λ and μ amended in turn.



Increasing either λ or μ has a very similar effect on the network, making the clustering coefficient larger and reducing the density, but also increasing the mean shortest path Figure 2. This is due to the agent’s potential locus of travel becoming smaller, which leads to more network-based meetings of agents in similar areas of social space and fewer random meetings which would reduce the shortest path.

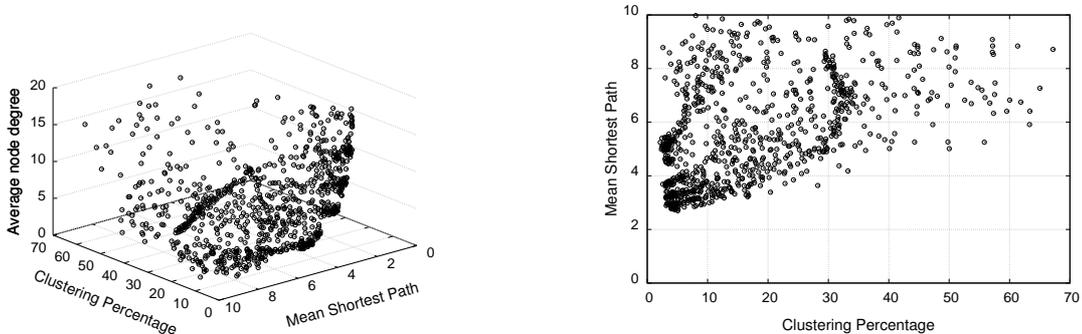
As μ can be interpreted as a “delayed” version of λ contingent on an agent having already formed social links, combining a small λ with large μ allows a combination of random meetings followed by mainly network-based meetings. This creates a network with a small mean shortest path, high clustering and a low density, which is not achievable using a single parameter in isolation.

It should be noted that increasing μ only increases the clustering up to a certain point, after which it remains relatively constant.

4.4 Coverage of the small-world space

Now that we have analysed how each parameter influences the topology of the network, we can investigate how much of the small-world space can be covered using our localised interactions model. We test a selection of networks across the parameter space, but exclude those with a mean shortest path above 10 or a density above 0.2, as we do not classify these as small-world [16]. However given that the expected clustering coefficient for a random graph can be approximated as $\frac{k}{N}$, where k is the average node degree and

FIG. 3. Coverage of the small-world space for node ageing. Networks are drawn from within the following parameter space: $2000 < M < 10000$, $0 < \alpha < 2$, $0.0005 < \lambda < 0.1$, $0 < \mu < 3$, $\beta = 1$.



N is the number of nodes, all parameter combinations yield networks with significantly more clustering than a random graph and so we do not need to eliminate any for that reason.

Although our model covers substantially more of the small-world space than other network growth models [14], there is still a tension between the mean shortest path and the clustering coefficient (Figure 3). While it is possible to keep the shortest path around 4 for clustering up to 30%, networks with a very high degree of clustering (60-70%) have a shortest path of above 6. This is an unavoidable consequence of the balance between network-based and random meetings, with extremely high clustering requiring that agents only operate within a much smaller area of the social space, thus greatly reducing random meetings and increasing the shortest path. However, the even distribution of points on the z axis of the left hand graph of Figure 3 shows that the networks can be controlled for density as required.

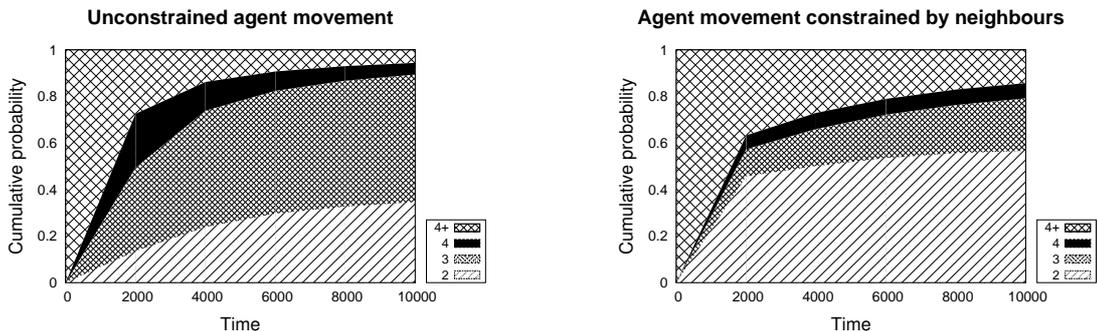
A similar analysis was undertaken for when network edges are aged rather than the nodes themselves. The results are similar in terms of parameter effect, but there is not such a broad coverage of the social space, as clustering over 35% is much harder to generate. The precise reasons for this are a potential area of further investigation.

4.5 Distribution of random to network-based meetings

In terms of the dynamism of the networks formed by our model, we have demonstrated the range of parameters for which global network statistics can be maintained and for which the networks remain stable. However, it would also be instructive to understand how new connections are being made, in terms of whether they are random or network-based. We can gauge this ratio by calculating the shortest path between the two nodes for each new edge formed (not including via the connection itself). A path length of 2 will indicate network-based “friend-of-friend” meetings, while 4 or above (including infinity if the two agents are not linked at all) will indicate a random meeting. A path length of 3 falls somewhere between a random and a network-based meeting.

We will analyse the time evolution of two networks based on extreme parameter settings and with edge ageing (Figure 4). One is set up for completely random movement of the agents with no dependence on existing links, while the other will form tight clusters by limiting the range that an agent can travel as it gains more neighbours.

FIG. 4. Distribution of the shortest path between new edges over time. The left hand graph uses the following parameters: $\alpha=0$, $\beta=1$, $\lambda=0.001$, $\mu=0$, $M=10,000$; with α and μ set to zero, agent movement is unconstrained by existing links. The right hand graph has parameters $\alpha=1$, $\beta=1$, $\lambda=0.01$, $\mu=3$, $M=10,000$; this combination of λ and μ limits agent movement based on existing links, leading to tight, modular clusters forming. The graphs show how the probability of the shortest path between two newly connected nodes being 2, 3, 4 or more than 4 varies over 10,000 timesteps.



Both start with random meetings as the network begins to form, but the distributions diverge rapidly after that. The network with unconstrained movement has a mean shortest path of around 2.15, so a path length of 3 or more between newly connected agents indicates a random meeting. This gives a ratio of random to network-based meetings of

around 2:1 after 10,000 timesteps (Figure 4, left panel).

By contrast, the network with constrained movement has a mean shortest path of 4. This indicates that the majority of new edges are formed by network-based meetings, with a ratio of random to network-based meetings of around 1:4 after 10,000 timesteps (Figure 4, right panel).

Therefore even though networks formed by extreme parameter combinations become biased towards either random or network-based meetings, they still maintain a reasonable probability of forming new links using both methods, which is analogous to real-world networks and corroborates our methodology of localised interactions.

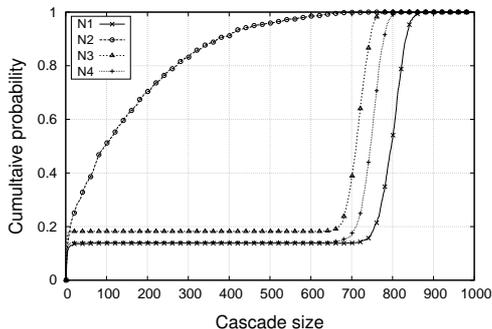
4.6 Network dynamics for an SIR model

In order to demonstrate how our network formation methodology can be incorporated into an agent-based model, and also to assess the variation in dynamical processes across the set of small-world networks, we implement the well-established SIR epidemiology contagion model on four networks developed using localised interactions. In the SIR model agents can take one of three states: *Susceptible*, *Infected* and *Recovered*. All agents begin in the *S* state, until at a given time, two seed agents within the population are switched to state *I*. On each subsequent timestep, an agent in state *I* has a probability γ of infecting a neighbour along any particular edge. In addition, any agent in state *I* has a probability ρ of moving to state *R*, which is an attractor (as agents can no longer move to states *I* or *S*). This model is well studied from the perspective of epidemic speed and topological variation on static networks, but there are relatively few results for dynamic networks.

We initialise all agents to be in the *S* state, then move two agents to state *I* after 2,000 timesteps, when the network is mature in terms of its characteristics. We set $\gamma=0.07$ and $\rho=0.2$, with these probabilities matching those used by Thiriot [14] in a similar experiment on the small-world space. The main difference with our simulation

is that the networks are dynamic. All four of our networks have a density of around 0.075 to control for that as a variable. In order to compare networks, we analyse the cascade size, which is defined as the total number of nodes that become infected before the epidemic dies out.

FIG. 5. Distribution of cascade sizes for an SIR process across four different networks, indicated by different line markers. The parameters used to generate each network are shown in the right-hand table. The cumulative probability of different cascade sizes is calculated from 1,000 instantiations of each network using the specified parameters.



Network	α	β	λ	μ	M
N1	0	1	0.01	1	10,000
N2	1	1	0.006	3	20,000
N3	0	1	0.001	0	1,800
N4	1	1	0.001	5	2,000

Three of the four networks, N1, N3 and N4, have a clear two-mode regime, with the epidemic spreading to either 80-90% of the population or staying confined to less than 5% (Figure 4). These three networks have clustering ranging from 0.02 for N3 up to 0.32 for N1, and similar shortest paths between 3.5 and 5.

However, the two-mode regime disappears for network N2 to be replaced by a much more regular distribution of cascade sizes, despite the network having only a slightly higher clustering of 0.37 and a slightly longer mean shortest path of 7.1. This suggests that there may be critical values for clustering and shortest path which lead to this transition, or that there are other important “hidden” network statistics which drive the spread of contagion.

The important result here is that although all four networks have an *S score* of at least 3 and so are classified as small-world [13], their probability distributions of cascade size are dramatically different. This indicates that assigning a generic conclusion regarding a dynamical process to be a result a small-world topology is problematic, and that more granularity within the small-world space is required.

5 Conclusion

We have developed a network formation model that uses plausible, localised agent interactions to create networks which are able to maintain key global network statistics, whilst also dynamically forming and breaking connections. We show that by altering up to five parameters controlling agent movement, networks can be created which display values for clustering, mean shortest path and density across the majority of the small-world space. These can be embedded into agent-based models which require specific network properties but also need agent relationships to evolve over time and within a given social space. Networks evolve through new link formation as a weighted balance between random and “friend-of-friend” meetings, and the death of existing nodes or edges after a certain time period.

Finally, we use our model to evolve four networks with different global statistics, but all classified as small-world. By demonstrating that the networks produce markedly different contagion dynamics for an epidemiological SIR process, we conclude that results cannot be generalised across small-world topologies. Thus, care must be taken when using a generic small-world network within an agent-based model.

October 7, 2013

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