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Regulating Unverifiable Quality by Fixed-Price Contracts

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Regulating Unverifiable Quality by Fixed-Price Contracts*

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Abstract

We apply the idea of relational contracting to a simple problem of regulating a single-product monopoly with unverifiable (then ex ante not contractible) quality. We model the interaction between the regulator and the firm as an infinitely repeated game; we observe that there exist self-enforcing contracts in which the regulator, using her discretionary power on the price (the contractible variable) can induce the firm to produce the required quality level by leaving it a positive rent. When players use grim trigger strategies, the optimal self-enforcing contract implies a distortion from the second best which is greater the more impatient is the firm and the larger is the effect of the price on the deviation profits. Whenever the equilibrium profits of the static game are strictly positive, even if the firm were infinitely patient, the optimal contract would not reach the second-best: it would ensure a quality-adjusted Ramsey condition and, at the same time, leave positive profits to the firm. We extend the model in a few ways: we find that when players use stick-and-carrot strategies, with an infinitely patient firm the second-best outcome is reached even if this implies to punish the deviating firm with negative profits. When instead the regulator is unable to perfectly monitor the firm's quality choice, the price/quality pair giving the highest payoff to the regulator does not directly depend on the firm's discount factor, which instead affects the probability of punishment. Our results suggest that, in fixed price regulatory contracts, the regulatory lag should be shorter the more relevant is the issue of unverifiability, in order to reduce the reward for opportunistic behavior by the firm.

KEYWORDS: quality regulation, relational contracts

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1 Introduction

How to regulate quality is an extremely sensitive issue which has been widely explored since the early days of the economics of regulation (Spence, 1975, Sheshinsky, 1976). Quality has many distinctive features: for instance, it may be difficult to observe, it has non-deterministic components, consumers' preferences may be hard to observe. In this paper we focus on unverifiability: unverifiability occurs whenever a variable, albeit observable by the parties, cannot be proven in front of a court and, as a consequence, cannot be contracted upon. In regulated industries it is often the case that some quality dimensions of the regulated firm's output are not verifiable: possible examples are courtesy and effectiveness of the customer service on technical and commercial issues, voltage of electricity or actual bandwidth provided in a particular moment in time, noise of a call, and so forth (see, for instance, Sappington, 2005).

The regulatory instruments commonly in use to regulate quality, such as quality standards and links between the quality provided and allowed revenues or prices, are able to influence those quality dimensions which are readily verifiable (Waddams Price *et al.*, 2008; De Fraja and Iozzi, 2008) but they may turn out not to be effective when quality is not verifiable. In this paper we suggest a way of regulating unverifiable quality which is based on the idea of *relational contracts* (Hviid, 2000; Levin, 2003; MacLeod, 2007).

Relational contracts are informal agreements and unwritten codes of conduct that are sustained by the value of future relationships, and are applicable even in the cases when the outcome of a relationship is based on some unverifiable variables. These agreements are typically sustained by the discretionary power that one of the parties has on some court-enforceable contract terms. This type of contracts fits in naturally with the nature of the interaction between the regulator and the regulated firm. These two parties typically interact repeatedly over time, and are both well informed on many variables affecting the outcome of the relationship, even though at least part of this knowledge cannot be used (or it is very expensive to use) in contracts or in front of a court. Also, the regulator enjoys quite a large discretion in making the decisions which affect the outcome of the relationship (Cowan, 2002).[1]

[1] Beesley and Littlechild (1989) clearly illustrates this point. On p 458, they state that "when setting X [which, under price cap regulation, is the annual rate of allowed price change, i.e. the parameter limiting the price level and dynamics (our note)] initially there are many degrees of freedom. [...] There is nothing unique, optimal, or mechanical about the initial choice of X. When X is reset, there are significantly fewer degrees of freedom.

We analyse a very simple problem of regulating a single-product monopolistic firm when there is full information by all parties and the regulatory instrument is a fixed-price contract; as described above, quality is endogenous and observable, but not verifiable. We model the interaction between the regulator and the firm as an infinitely repeated game in which the regulated price can be used by the regulator as an incentive device to enforce the mandated level of quality. We observe that, under grim trigger strategies, provided both players are sufficiently patient, there exist self-enforcing regulatory contracts in which the firm prefers to produce the quality mandated by the regulator, while the regulator chooses to leave the firm a positive rent as a reward to its quality choice. We then proceed to characterise the self-enforcing contract which maximises the regulator's objective function. We show that, under normal circumstances, this contract implies a distortion from the second best. This distortion is greater the more impatient is the firm and the larger is the (marginal) effect of the contractual price on the profits the firm would make by deviating from the offered contract. Whenever the equilibrium of the stage game entails positive profits, even if the firm were infinitely patient, the optimal contract would grant positive profits to the firm; however, it would satisfy the typical Ramsey condition of tangency between the isowelfare and the isoprofit. We also show that, at the optimal contract, the distortion (relatively to the second best) only affects the verifiable variable, i.e., the price: the mandated (unverifiable) quality always obeys a second best condition. To further illustrate this price distortion, we show that, under standard assumptions on the nature of the consumers' and regulator's preferences, the price in the optimal contract obeys a Ramsey-like inverse elasticity rule; the optimal mark-up depends not only on the demand elasticity, but also on the discount factor and the marginal effect of price on both deviation profits and social welfare.

To check for the robustness of our result, we extend our analysis in three different directions. First, we allow players to use stick-and-carrot strategies, which clearly allow harsher punishments in case of deviation. In this setting, we find that, with an infinitely patient firm, there exist contracts that are able to enforce the second best. These, however, entail negative punishment profits. Indeed, were the regulator unable to induce negative profits to the firm in some periods (as it is normally posited in most of the regulation literature), the impossibility to reach the second-best would also carry over to this case in which the regulator uses a more "sophisticated" type of strategy. Second, we investigate the case of a regulator being unable to perfectly monitor the

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Nevertheless, there invariably *are* degrees of freedom open to the regulator". Littlechild was the Director General of OFFER, the UK energy regulator, from 1989 to 1998.

firm's quality choice. The socially optimal probability of punishment increases with the deviation profits, while it is decreasing in the discount factor and the contractual profits. The contract giving the regulator the highest payoff implies a distortion with respect to the second best dependent only on the single-period gain the firms obtains from the deviation. Third, we analyse the case of quality having two dimensions, one verifiable and one not, and confirm that the optimal contract entails a distortion only on verifiable variables (here, price and one of the quality components).

Our contribution is clearly related to the large literature on quality regulation (Spence, 1975; Sheshinsky, 1976 and, more recently, Weisman, 2005; Currier, 2007; De Fraja and Iozzi, 2008 and Auray *et al.*, 2011), where, however, the issue of unverifiability arises only in few papers (see, for instance, Lewis and Sappington, 1992; Laffont and Tirole, 1993 and Dalen, 1997); to the best of our knowledge, this is the first attempt of dealing with relational contracts within a framework of pricing regulation with unverifiable quality. This paper makes a contribution also to the issue of optimal regulatory lag (Armstrong *et al.*, 1995), when one interprets the discount factor as resulting from the frequency of interactions between the players. Our result suggests that the frequency of the price revisions should be higher the more relevant is the issue of unverifiability. This is because a shorter regulatory lag reduces the reward for opportunistic behaviour by the firm and allows the regulator to elicit "better" price and quality pairs. A similar indication comes also in the case of procurement contracts, which our paper contributes to study once one focuses on the case of repeated purchases and reads the discount factor as depending also on the probability of continuation of the contract: our results suggest that a shorter contract length and some form of favouritism towards the incumbent firm when re-tendering (future) contracts increases the value of adhering to the offered (current) contract, and results in a price and quality pair which is closer to the one most socially preferred.[2]

The rest of the paper is organised as follows. Section 2 presents the model. The equilibria of the static and the dynamic game in our basic setting are characterized in Section 3. Section 4 contains some extensions of our analysis and robustness checks of our results. Section 5 concludes.

[2]Related papers on repeated procurement with non-contractible elements are Albano *et al.* (2011), Calzolari and Spagnolo (2009), Doni (2006), Iossa and Rey (2010), Kim (1998), Kremer *et al.* (2006) and Sasaki and Strausz (2008).

2 The Basic Model

We analyse an infinite horizon game in which two parties, a regulator and a monopolistic firm, interact at dates $t = 0, 1, \ldots, \infty$. Let δ be the discount factor, common to the firm and the regulator.

The monopolist produces one good, whose demand is given by $x(p, q)$, where p denotes the price of the good and q its quality; we assume that $p \in \mathbb{R}_+$ and $q \in Q \equiv [q^-, q^+] \subseteq \mathbb{R}_{++}$. The demand function is continuous and twice differentiable (this applies to all other functions we introduce in this Section). Demand satisfies $x_p(p, q) < 0$ and $x_q(p, q) > 0$ whenever $x(p, q) > 0$, where subscripts denote partial derivatives.

The firm's technology is described by the cost function $c(x, q)$ which satisfies, plausibly, $c_x(x, q) > 0$ and $c_q(x, q) > 0$. To avoid corner solutions, we assume that $\lim_{q \to q^-} c_q(x, q) = 0$ and that $\lim_{q \to q^+} c(x, q) = +\infty$: a marginal increase of quality is costless when quality is at its minimum and is infinitely costly when quality is maximal. The firm's profits are therefore given by $\pi(p, q) = x(p, q)p - c(x(p, q), q)$.

The regulator's objective function is given by $v(p, q)$ which is simply assumed to satisfy the following conditions: $v_p(p, q) \leq 0$, and $v_q(p, q) > 0$.[3]

In most of the paper, we analyse a dynamic game which is an infinite repetition of the following sequential stage game:

1. the regulator makes an offer $\mathcal{F} \equiv \{p', q'\}$ in which she asks the firm to produce a good of quality q' and sets the market price p' at which the good has to be sold;
2. the firm chooses whether or not to accept the contract: if the firm accepts the contract, the game proceeds to the following stage; otherwise, it does not produce, reservation payoffs $\tilde{\pi}$ and $\tilde{V}$ are collected and the game ends;
3. the firm chooses the actual quality level q''; at the end of this stage the regulator observes q'', and the payoffs $v(p', q'')$ and $\pi(p', q'')$ are realized.

We assume that the reservation value for the regulator is low enough to make her always prefer that the firm undertakes production in each period. This is a standard hypothesis in the regulation literature, motivated by the specific

[3] At this stage of the paper, we purposely do not impose any further restrictions on the consumers' and regulator's preferences. The analysis then applies to the standard case of an utilitarian benevolent regulator (as it is the case in Section 3.3) but it is also valid, for instance, when the regulator has some distributional (Feldstein, 1972 and, for an application to price cap regulation Iozzi *et al.*, 2002, and Valentini, 2006) or environmental concerns (Oates and Portney, 2003), or she is captured by the regulated firm (Stigler, 1971).

nature of the industries subject to regulation. Its immediate consequence is that the second stage of the game can be safely ignored in the rest of the analysis, since the firm will never choose to reject the offer and produce a null quantity for one period.

We analyse a repeated game of perfect monitoring. However, despite the realisation of price and quality is fully observable by both players, quality is not enforceable in a court of law. Then, the regulator cannot impose any directly enforceable penalty on the firm if she observes $q' \neq q''$.

Before proceeding into the analysis of the game, we state:

Definition 1. *Let the following definitions hold:*

a) for any price p, *let* $\widehat{q}(p) \equiv \arg\max_{q} \pi(p,q)$ *and* $\widehat{\pi}(p) \equiv \pi(p, \widehat{q}(p))$;

b) let p^0 *be the smallest price such that* $\widehat{\pi}(p^0) = 0$.

In words, $\widehat{q}(p)$ is the quality level that delivers the highest profit to the firm for any possible price. We assume it exists and is unique. Similarly, $\widehat{\pi}(p)$ is the profit the firm can make, for any given price, when it optimally chooses its quality. Also, p^0 is the price which ensures that the firm obtains zero profits when it freely chooses its quality level, given this price. If more than one such price exist, p^0 denotes the smallest, that is, the one that gives the highest value of $v(p, \widehat{q}(p))$ amongst those satisfying the zero-profit condition stated in the definition.

We also state:

Definition 2. *Let* p^R *and* q^R *be the pair of price and quality which solves the following problem:*

$$\max_{p,q} v(p,q) \quad \text{s.t. } \pi(p,q) \geq 0 \tag{1}$$

In words, p^R and q^R is the Ramsey price and quality pair which maximises the (static) regulator's objective function, subject to a non-negativity constraint on the firm's profits. It is easy to show that, at p^R and q^R, the following holds:

$$\frac{v_p\left(p^R, q^R\right)}{v_q\left(p^R, q^R\right)} = \frac{\pi_p\left(p^R, q^R\right)}{\pi_q\left(p^R, q^R\right)}, \tag{2}$$

and that, at p^R and q^R, the non-negativity constraint holds as an equality. We assume p^R and q^R to exist, be unique and different from p^M and q^M, where p^M and q^M are the profit maximising price and quality.

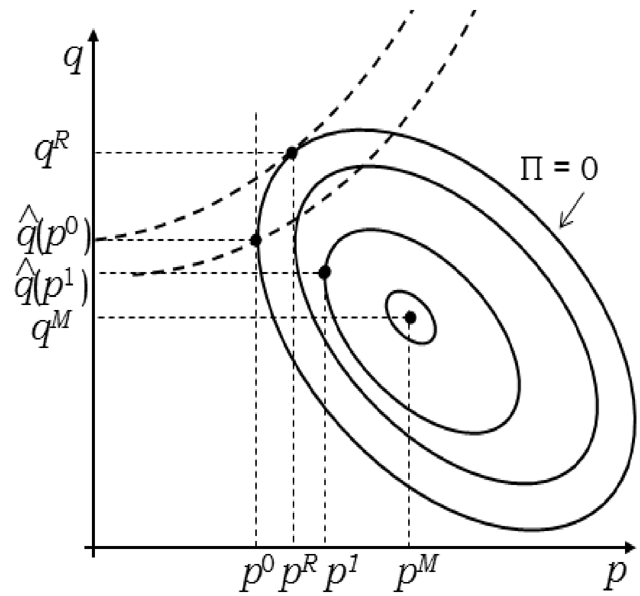

Figure 1: The static game.

Figure 1 illustrates these Definitions. It depicts the price-quality cartesian plane; the solid curves are the isoprofit lines and the dashed curves are the isowelfare lines, upward sloping because welfare is increasing in quality and decreasing in price.[4] The pair $\{p^M, q^M\}$ is the unconstrained profit maximising price and quality pair, and $\{p^R, q^R\}$ is the second best pair: at this point, the zero-profit isoprofit line is tangent to the isowelfare map. At prices p^0 and p^1, the firm, when freely choosing its quality level, selects $\widehat{q}(p^0)$ and, respectively, $\widehat{q}(p^1)$: at the price and quality pair $\{p^0, \widehat{q}(p^0)\}$ the firm makes zero profits.

[4]In the Figure, isowelfare lines are drawn under the hypothesis that the $v(p, q)$ is quasi-concave. This needs not be the case and all our results are valid even when $v(.)$ has a different nature. With consumers with quasi-linear preferences and a benevolent regulator (as in Section 3.3), quasi-concavity of $v(p, q)$ would reflect the quite natural (but not always satisfied) property of consumers' preferences that their willingness to pay for increases in quality is higher when quality is low than when quality is already high (for further discussion, see De Fraja and Iozzi, 2008).

3 Equilibrium

3.1 The static game

This Subsection characterises the equilibrium of the stage game. This is interesting not only in itself, but also as its features will play an important role in the characterisation of the equilibrium of the dynamic game.

In the static game, if quality is not verifiable, the regulator cannot enforce the second best quality level. Since the regulator can only observe but not punish any choice of quality other than the mandated level, we are back in the context of price regulation with endogenous quality, firstly analysed by Spence (1975). It is then straightforward to characterize the equilibrium of the stage game described above. In the last stage of the game, for any price mandated by the regulator, the firm chooses the profit maximizing quality level $\widehat{q}(p)$. Anticipating this, in the first stage the regulator makes an offer $\mathcal{F}^S \equiv \left\{p^S, \widehat{q}(p^S)\right\}$, where p^S comes as the solution of the following problem:[5]

$$\max_p v(p,q) \tag{3}$$
$$\text{s.t. } \pi(p,q) \geq 0$$
$$q = \widehat{q}(p).$$

The properties of this equilibrium price are described in the following Proposition:

Proposition 1 *The equilibrium price p^S offered by the regulator in the stage game has the following features:*

- $p^S = p^0$ *whenever* $-v_p\left(p^S, \widehat{q}\left(p^S\right)\right) > v_q\left(p^S, \widehat{q}\left(p^S\right)\right)\widehat{q}_p\left(p^S\right)$, *which implies* $\widehat{\pi}\left(p^S\right) = 0$, *and*
- $p^S > p^0$ *whenever* $-v_p\left(p^S, \widehat{q}\left(p^S\right)\right) = v_q\left(p^S, \widehat{q}\left(p^S\right)\right)\widehat{q}_p\left(p^S\right)$, *which implies* $\widehat{\pi}(p^S) > 0$.

Proof To solve problem (3), we set up the Lagrangean incorporating the second constraint

$$L = v(p, \widehat{q}(p)) + \lambda\widehat{\pi}(p).$$

[5] We take $\widehat{q}\left(p^S\right)$ as the quality level included in the offer $\mathcal{F}^S$ only for the sake of definiteness; indeed, any quality level could be part of this offer because, in this static setting, the regulator anticipates that the firm will always choose its profit maximizing quality level and that she cannot prevent or punish this behaviour.

FOCs are:

$$L_p = v_p\left(p^S, \widehat{q}\left(p^S\right)\right) + v_q\left(p^S, \widehat{q}\left(p^S\right)\right)\widehat{q}_p\left(p^S\right) + \lambda\widehat{\pi}_p\left(p^S\right) = 0;$$
$$L_\lambda = \widehat{\pi}\left(p^S\right) \geq 0; \qquad \lambda \geq 0; \qquad \lambda\widehat{\pi}\left(p^S\right) = 0.$$

If $\lambda = 0$, then $\widehat{\pi}\left(p^S\right) \geq 0$ and $v_p\left(p^S, \widehat{q}_p\left(p^S\right)\right) + v_q\left(p^S, \widehat{q}\left(p^S\right)\right)\widehat{q}_p\left(p^S\right) = 0$. Instead, if $\lambda > 0$, then $\widehat{\pi}\left(p^S\right) = 0$ and $v_p\left(p^S, \widehat{q}_p\left(p^S\right)\right) + v_q\left(p^S, \widehat{q}\left(p^S\right)\right)\widehat{q}_p\left(p^S\right) + \lambda\widehat{\pi}_p\left(p^S\right) = 0$. Since $\widehat{\pi}(p)$ is monotonically increasing in p whenever $p < p^M$, this establishes the result. ■

Proposition 1 illustrates that the optimal static offer is such that the firm may make strictly positive profits.[6] When the optimal offer implies strictly positive profits, the non-negativity constraint is slack; the regulator sets the price to equalize the (negative) marginal direct effect on welfare with the (positive) marginal indirect effect, due to an increase in the quality provision – i.e. $-v_p\left(p^S, \widehat{q}(p^S)\right) = v_q\left(p^S, \widehat{q}(p^S)\right)\widehat{q}_p\left(p^S\right)$. Notice that, in this case, a marginal increase in the price necessarily induces an increase in the quality provided by the firm. On the other hand, when the optimal offer implies zero profits, the regulator would prefer to reduce the price because this has a direct effect on social welfare which outplays the indirect effect caused by the induced change in quality; however, the non-negativity constraint on the firm's profits limits a further price reduction. Notice that in this case, at the equilibrium, the sign of the marginal change in quality due to a marginal price change – i.e. $\widehat{q}_p\left(p^S\right)$ – is indeterminate.

The equilibrium of the stage game is illustrated in Figure 2. In both panels, the locus aa' illustrates the optimal quality choices for the different price levels, i.e. $\widehat{q}(p)$. Taking this as a constraint, the regulator chooses her optimal one-shot price p^S to maximise social welfare. Depending on the local relative slope of iso-welfare, $-\frac{v_p\left(p^S,\widehat{q}\left(p^S\right)\right)}{v_q\left(p^S,\widehat{q}\left(p^S\right)\right)}$, and of $\widehat{q}(p^S)$, the optimal price may be given by a tangency condition between the isowelfare and the locus aa', as in the left panel, or may be a corner solution, as in the right panel. Clearly, given the many possible shapes the locus aa' can take on, restrictions are necessary to ensure that the solution to the regulator's problem is unique or, more restrictively, exists altogether.

In the Figure, the grey shaded areas illustrate the price-quality combinations which are a Pareto improvement relatively to the static equilibrium

[6] Note that the solution to problem (3) need not be unique. In case of multiple solutions, for reasons that will be clearer thereafter, we select p^S as the solution giving the firm the lowest profit. Also, to avoid unrewarding solutions, throughout the paper we assume $\widehat{\pi}_p\left(p^S\right)$ to be strictly positive.

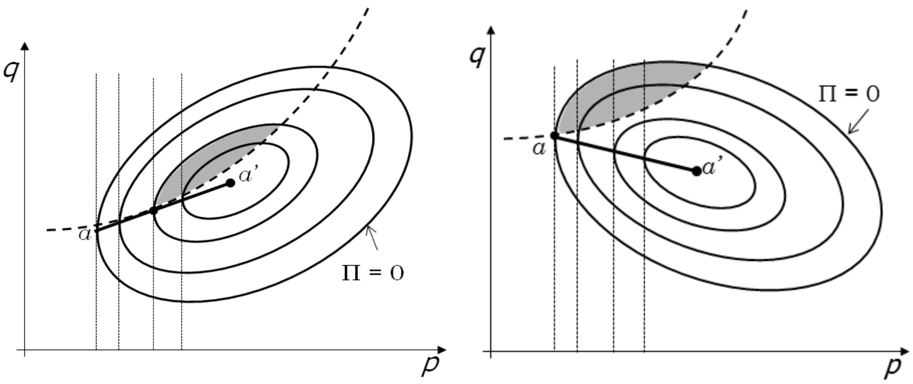

Figure 2: The optimal static contract.

and can therefore be contracted on relationally. The nature of these contracts and the characteristics of the optimal contract are discussed in the next Subsection.

3.2 The dynamic game

In this Subsection, we study the dynamic game, given by an infinite repetition of the sequential stage game just discussed. We illustrate that a relational contract ensures that both players obtain a payoff higher than the one they would get in a static setting and, assuming that players use grim trigger strategies, show the nature of the price and quality pair which are part of the contract that maximises the regulator's objective function.

With unverifiable quality, a regulatory relational contract is a strategy profile such that in each period the regulator makes an offer $\mathcal{F}^C \equiv \{p^C, q^C\}$ and the firm chooses to adhere to this offer. This regulatory relational contract is self-enforcing if the strategy profile is a perfect equilibrium of the repeated game.

A game of this sort has multiple equilibria. Indeed, the Folk theorem ensures that any pair of payoffs which are feasible and individually rational are the payoffs of some subgame perfect equilibrium, provided that δ is sufficiently high.[7] We concentrate on equilibria resulting from the adoption of the

[7] Sorin (1995) proves that the Folk theorem, proved by Fudenberg and Maskin (1986) for simultaneous repeated games, also applies to sequential repeated games provided that full dimensionality condition (FDC) holds. This requires that the convex hull of the set of the feasible payoff vectors of the stage game must have dimension equal to the number of players, or equivalently a nonempty interior. FDC is clearly satisfied in our model. Abreu *et al.* (1994) and Wen (1994, 2002) further weaken these requirements.

following grim trigger strategies:

- **regulator**: the regulator begins the game by making to the firm an offer $\mathcal{F}^C$. In the following repetitions of the game, she keeps making this offer as long as the history of the game is such that no price/quality pair different from $\{p^C, q^C\}$ was previously chosen; otherwise, she reverts indefinitely to her equilibrium strategy in the stage game;
- **firm**: the firm chooses the quality q^C as long as the history of the game is such that no price or quality different from p^C and q^C was previously chosen; otherwise, it reverts indefinitely to its equilibrium strategy in the stage game.

Notice the somehow different nature of the two strategies, due to the sequential nature of the stage game; while the regulator may react to a quality level different from q^C only in the following period this choice has been made, an offer different from $\mathcal{F}^C$ offered by the regulator is immediately observed by the firm and triggers the firm's reaction in the same period it is made.

Our pair of trigger strategies forms a subgame perfect equilibrium of our game, for an adequate choice of $\mathcal{F}^C$ (so that the per period equilibrium payoff for each player exceeds its equilibrium payoff in the stage game) and for sufficiently patient parties (δ is close enough to 1). Subgame perfection is ensured by the absence of profitable one-shot deviations (Mailath and Samuelson, 2006), where a one-shot deviation from strategy σ is a strategy which, at time t, prescribes a different action than σ for a unique history, but which plays identical to σ in every period other than t.

Clearly, there are many offers $\mathcal{F}^C$ supporting a trigger strategies equilibrium; our next aim is to characterise the offer which is preferred by the regulator. We let $\mathcal{F}^* \equiv \{p^*, q^*\}$ be such an offer; it comes as the solution to the following problem:

$$\max_{p^C, q^C} \sum_{t=0}^{\infty} \delta^t v(p^C, q^C) = \frac{1}{1-\delta} v(p^C, q^C) \tag{4}$$

$$\text{s.t. } \frac{1}{1-\delta} \pi\left(p^C, q^C\right) \geq \widehat{\pi}\left(p^C\right) + \frac{\delta}{1-\delta} \widehat{\pi}\left(p^S\right) \tag{5}$$

where constraint (5) is the firm's incentive compatibility constraint (ICC, henceforth). The maximisation problem deserves two brief comments: first, an incentive compatibility constraint for the regulator is not needed, since any choice of the regulator different from the equilibrium contract is immediately observed and punished by the firm, and it is therefore not profitable for the regulator. Also, constraint (5) implies that the single-period participation constraint for the firm holds; this constraint often features dynamic regulatory problems of this kind.[8]

We can now state the main result of the Section:

Proposition 2 *When players use grim trigger strategies, the price and quality pair* $\{p^*, q^*\}$ *solving problem (4)-(5) has the following properties:*

- *it satisfies the following conditions:*

$$\frac{v_p(p^*,q^*)}{v_q(p^*,q^*)} = \frac{\pi_p(p^*,q^*) - (1-\delta)\widehat{\pi}_p(p^*)}{\pi_q(p^*,q^*)} \tag{6}$$

and

$$\pi(p^*,q^*) = (1-\delta)\widehat{\pi}(p^*) + \delta\widehat{\pi}\left(p^S\right); \tag{7}$$

- *the optimal contractual quality is always set to satisfy the static quality-adjusted Ramsey condition, given the contractual price.*

Proof The Lagrangian of problem (4)-(5) is:

$$L = \frac{1}{1-\delta} v\left(p^C, q^C\right) + \lambda\left[\frac{1}{1-\delta}\pi\left(p^C, q^C\right) - \widehat{\pi}(p^C) - \frac{\delta}{1-\delta}\widehat{\pi}(p^S)\right]. \tag{8}$$

FOCs are:

$$L_p = v_p(p^*,q^*) + \lambda\left[\pi_p(p^*,q^*) - (1-\delta)\widehat{\pi}_p(p^*)\right] = 0; \tag{9}$$

$$L_q = v_q(p^*,q^*) + \lambda\pi_q(p^*,q^*) = 0; \tag{10}$$

$$L_\lambda = \frac{1}{1-\delta}\pi(p^*,q^*) - \widehat{\pi}(p^*) - \frac{\delta}{1-\delta}\widehat{\pi}\left(p^S\right) \geq 0; \quad \lambda \geq 0; \quad \lambda L_\lambda = 0. \tag{11}$$

From (9) and (10), it follows that $\lambda > 0$; if this were not the case, we would have that $v_p(p^*,q^*) = v_q(p^*,q^*) = 0$, which clearly contradicts the hypothesis that the first best is out of reach. Therefore, $L_\lambda = 0$ in (11), which gives (7). Also, dividing (9) by (10), we get (6). The second part of the Proposition

[8] Notice that we do not impose here a single period non-negativity constraint. Instead, the fact that the firm never gets non-negative profits follows directly from the fact that the punishment phase is nothing but an infinite repetition of the equilibrium of the static game; this implies that, in case of a deviation, the firm is punished with the (always non-negative) equilibrium profits of the static game. More technically, a single period non-negativity constraint is not assumed but it is implied by constraint (5) and the fact that $\widehat{\pi}\left(p^S\right) \geq 0$. For more on this, see the discussion at the end of Sect. 4.1.

is established by simply observing that (10) is identical to the corresponding FOC obtained solving problem (1). ■

Conditions (6) and (7) define the optimal equilibrium price and quality pair and illustrate the way it departs from the second best. Condition (6) differs from the quality-adjusted Ramsey condition (2) because of the second term in the numerator of the RHS; this depends, first, on the discount factor and, second, on the marginal effect on the deviation profits of a change in the contractual price. To interpret (6), note that, in the quality-adjusted Ramsey condition of tangency (2), the marginal rate of substitution between price and quality is equated between the regulator and the firm. On the contrary, here, the regulator sets a price lower than the one which would ensure the tangency between isoprofit and isowelfare. This is because it takes into account that the higher is the price offered, the higher are the profits the firm makes, at that price, if it fails to deliver the mandated quality. This, and the discount factor, which tells how "impatient" is the firm, determines the change in the firm's incentive to deviate following a change in the price. The greater is this incentive the smaller is the regulator's willingness to substitute away price with quality. Only if the firm were infinitely patient, the optimal contract would correspond to a tangency condition between the iso-welfare and the iso-profit, as described by the standard quality-adjusted Ramsey condition (2).

On the other hand, condition (7) determines the level of the profits the regulator has to ensure to the firm. These increase not only with the profits the firm obtains when it fails to deliver the mandated quality level – i.e. the "deviation" profits $\widehat{\pi}(p^*)$ –, but also with the profits the firm obtains in case of reversal to the static equilibrium – i.e. the "punishment" profits $\widehat{\pi}\left(p^S\right)$. When these latter profits are strictly positive, even if the firm were infinitely patient, also the optimal contract must ensure the firm strictly positive profits. In other words, as δ approaches 1, condition (6) approaches the quality-adjusted Ramsey condition (2), but this tangency condition would not occur on the zero-profit isoprofit.

Finally, the second part of Proposition 2 illustrates that only the price, and not the quality, is "distorted" relatively to the second best to give the firm incentives to cooperate and, at the same time, minimise the distortion. Quality is instead at a level that satisfies a second best condition. Notice, however, that this does not imply that the optimal contractual quality is set at the quality-adjusted Ramsey level; since typically $p^R \neq p^*$, this affects the second best condition giving the quality required by the regulator.

3.3 Relational Ramsey pricing

In this Subsection, we draw a closer connection between our results and the established benchmark of Ramsey prices. To ensure an immediate comparability with the extant literature, we use standard assumptions on the nature of consumers' and regulator's preferences and then reformulate our results.[9]

Assume that the firm's output is demanded by L consumers with quasi-linear indirect utility given by $\nu^\ell(p,q)+y^\ell$, $\ell=1,\ldots,L$, where y^ℓ is consumer ℓ's income. Because of the additive formulation, there are no income effects: individual ℓ's demand is simply given by $x^\ell(p,q)$, and aggregate demand is given by the sum of individual demands: $x(p,q)=\sum_\ell x^\ell(p,q)$. Assume also that the regulator is a benevolent utilitarian. Her objective function $v(p,q)$ is the unweighted sum of individuals' utility: $v(p,q)=\sum_\ell \nu^\ell(p,q)$, after normalising away total income. The quasi-linear nature of the individuals' preferences implies that Roy's identity can be written as $v_p(p,q)=-x(p,q)$.

Denoting with $\eta(p,q)$ the price elasticity of the demand for good x, we can then state the following Proposition:

Proposition 3 *Assume consumers have quasi-linear preferences and the regulator is a benevolent utilitarian. When players use grim trigger strategies, the optimal contractual price satisfies*

$$\frac{p^*-c_x(x(p^*,q^*),q^*)}{p^*}=\frac{1}{\eta(p^*,q^*)}\left[\frac{\lambda-1}{\lambda}-(1-\delta)\frac{\widehat{\pi}_p(p^*)}{x(p^*,q^*)}\right]. \tag{12}$$

The Proposition comes from the use of Roy's identity in the FOCs of problem (4)-(5) and a simple rearranging, and the proof is therefore omitted. It says that optimal contract entails a price which obeys a typical inverse–elasticity rule. The basic structure of the mark-up is very similar to the case of traditional Ramsey prices, apart for the obvious difference of a Lagrange multiplier giving now the shadow cost of the firm's incentive compatibility constraint, rather than the one of non-negativity constraint, as with the standard Ramsey case. The main novelty is that the mark-up is now affected also by the firm's incentive to deviate, as illustrated by the firm's discount factor and the derivative w.r.to profits of the deviation profits. The discount factor is positively related to the mark-up, illustrating that the more patient is the

[9] See the seminal contributions of Baumol and Bradford (1970) and Feldstein (1972), and the ensuing literature. See Bös (1981) for a comprehensive treatment of quasi-linearity of consumers' preferences and its consequences on the properties of the objective function of a benevolent regulator.

firm, the smaller is the firm's incentive to deviate and then the higher can be the contractual price. On the other hand, the price-cost margin is lower the higher is $\widehat{\pi}_p(p^*, q^*)$, that is the larger is the effect of a price increase on the gain the firm makes when deviating from the offered contract. Clearly, the mark-up is also negatively affected by the effects that the contractual price has on consumers' welfare, as given by $-v_p(p^*, q^*) = x(p^*, q^*)$.

3.4 Contract length and probability of continuation

Propositions 2 and 3 illustrate the importance of the discount factor in shaping the nature of the optimal contract: in particular, they illustrate that the optimal contract delivers an outcome which differs the more from the second best the smaller is the discount factor. To fully comprehend the implications of this result and to relate our results to the extant literature, it is useful to further discuss the nature of the discount factor. Indeed, standard textbook analysis shows that δ reflects not only the players' intertemporal preferences, but also other circumstances, such as frequency of interactions and the probability of continuation of the game in the following periods.

The frequency of interactions positively affects the value of the discount factor, since a high value of δ is associated to a low value of the per-period interest rate. In our setting, an increase in the frequency of interactions is equivalent to a reduction in the time between two successive price (and quality) reviews, something which is normally referred to as regulatory lag. The existing literature illustrates that, when the firm's cost is endogenous, there is a trade-off in setting the regulatory lag: the longer is the regulatory lag, the higher is the incentive the firm has to undertake cost-reducing efforts, but also the greater the allocative inefficiency arising from the firm's excessive profits (Armstrong *et al.*, 1994; and Armstrong *et al.*, 1995). Our results suggest the existence of a further determinant in setting the regulatory lag in regulatory settings in which unverifiability plays a role. Since our equilibrium outcome becomes closer to a quality-adjusted Ramsey solution as δ increases, a short regulatory lag increases the regulator's ability to induce a second best outcome. This is because a short regulatory lag causes small gains from the deviation from the offered contract, thereby reducing the need to increase the firm's profits to induce it to deliver the required quality.

On the other hand, our game is formally equivalent to another game of indeterminate length in which δ not only reflects the players' intertemporal preferences and the frequency of interactions, but also the probability of continuation of the game in the following periods. This allows to interpret our model as a model of dynamic regulation in which, at some moment in time, the incumbent firm may lose the contract or can be imposed a contract of different and unknown nature. In other words, our δ can be reinterpreted as a measure of the regulatory risk faced by the firm. Our results then show that the greater is this regulatory risk, the larger is the distance to the second best of the optimal regulatory relational contract.

The interpretation of the discount factor as incorporating the probability of continuation of the game allows also to use our model to analyse the case of procurement with repeated purchases, in which the incumbent firm may lose the contract to a rival, possibly because of some (here, unspecified) competitive tendering procedure. Our results on the frequency of interactions are then fully consistent with Calzolari and Spagnolo (2009) (see also Spagnolo, 2012), who analyse a model of repeated competitive procurement with unverifiable quality. They find that a reduction in the contract length (i.e., more frequent interactions), in increasing the probability of collusion (see also Kremer *et al.*, 2006, and Sasaki and Strausz, 2008), increases the value of the contract to the firm and, therefore, induces it to deliver a "good" level of the non-contractible variable.

Also, our paper illustrates that there is scope for some favouritism towards the incumbent firm when re-tendering the contract, which increases the probability of continuation of the long-term relationship with the incumbent. This increases the value of adhering to the regulator's offer and results in a price and quality pair which is closer to the second best. Our results are then in line with Laffont and Tirole (1988), who analyse a two-period model of natural monopoly and second-sourcing. They find that, when cost-saving investment are transferable but not observable, the regulated firm has insufficient incentive to invest since its investment may benefit the rival in the following period. Then, in the auction to choose the contractor in the second stage of the game, the regulator may find it optimal to favour the incumbent to increase its incentive to invest in the first stage.

4 Extensions and robustness checks

In this Subsection, we consider three different extensions to our basic model. We start by assuming that players use stick-and-carrot strategies, rather than grim trigger strategy. Second, we look at the case of the regulator being unable to perfectly monitor the firm's quality choice. Last, we investigate the case in which the quality of the good produced by the regulated firm has two different dimensions, one verifiable and one unverifiable.[10]

4.1 Stick and carrot strategies

In this Section, we characterise the equilibrium resulting from players using stick-and-carrot strategies. In the previous Section, the use of grim trigger strategies constrained the punishment payoff to be no lower than the static game equilibrium payoff; this limitation was a potential explanation of the impossibility to reach the second best even with an infinitely patient firm. Therefore, in this Subsection we explore whether this impossibility can be overcome by allowing harsher punishments, as it is indeed the case with stick-and-carrot strategies. These strategies are also somewhat more palatable in a regulatory context, since they allow for a punishment of limited length, avoiding the unrealistic possibility of an everlasting punishment.

We concentrate on players adopting the following stick-and-carrot strategies:[11]

- **regulator**: the regulator begins the game by making an offer $\mathcal{F}^C \equiv \{p^C, q^C\}$ and keeps making this offer as long as the history of the game is such that no price/quality pair different from $\{p^C, q^C\}$ was previously chosen. Alternatively, the regulator offers $\mathcal{F}^P \equiv \{p^P, q^P\}$ for T periods; if during these T periods the price/quality pair $\{p^P, q^P\}$ is observed, in period $T+1$ the regulator reverts to the initial choice of $\mathcal{F}^C$. Otherwise, if during any of these T periods any price/quality pair different from $\{p^P, q^P\}$ was chosen, the regulator offers again $\mathcal{F}^P$ for the next T periods;
- **firm**: the firm chooses quality q^C as long as the history of the game is such that no price and quality different from p^C and q^C was previously chosen. Alternatively, if it observes a price different from p^C, the firm chooses $\widehat{q}(p)$ in the same period and then q^P for the following T periods; if during these T periods no price and quality different from p^P and q^P is chosen, in period $T+1$ the firm reverts to the initial choice of q^C. Otherwise, if during any of these T periods a price different from p^P is chosen, the firm first chooses $\widehat{q}(p)$ in the same period and then q^P for the following T periods.

[10]In these Section, we only present the assumptions of the model that vary with respect to the basic model.

[11]See Abreu (1986) and (1988), and Abreu *et al.* (1986) for a formal discussion.

To analyse our problem, we first need to introduce some further notation. Let Π^P be the present value of firm's future profits during and after a phase of punishment for a deviation, assuming that no deviation occurs during the T periods of the punishment phase so that, after this phase, the "cooperative" profits are re-established. Formally, it is given by

$$\begin{aligned}\Pi^P &\equiv \sum_{t=0}^{T-1} \delta^t \pi\left(p^P, q^P\right) + \sum_{t=T}^{\infty} \delta^t \pi\left(p^C, q^C\right) \\ &= \frac{1-\delta^T}{1-\delta} \pi\left(p^P, q^P\right) + \frac{\delta^T}{1-\delta} \pi\left(p^C, q^C\right). \end{aligned} \tag{13}$$

We also let Π^{DP} be the present value of firm's future profits following a deviation from the punishment phase. Formally, it is given by

$$\Pi^{DP} \equiv \widehat{\pi}\left(p^P\right) + \delta \Pi^P. \tag{14}$$

We can then write the incentive compatibility constraint, which ensures that the firm prefers to adhere to the contract, in the following recursive way

$$\frac{1}{1-\delta} \pi\left(p^C, q^C\right) \geq \widehat{\pi}(p) + \delta \Pi^P; \tag{15}$$

also, the constraint ensuring that the firm does not have an incentive to deviate from the punishment path takes the following form[12]

$$\Pi^P \geq \Pi^{DP}. \tag{16}$$

Similarly, on the regulator's side, let V^P be the present value of the regulator's future payoffs during and after a phase of punishment for a deviation, assuming that no deviation occurs during the T periods of the punishment phase so that, after this phase, the "cooperative" payoffs are re-established. Formally, it is given by

$$\begin{aligned}V^P &\equiv \sum_{t=0}^{T-1} \delta^t v\left(p^P, q^P\right) + \sum_{t=T}^{\infty} \delta^t v\left(p^C, q^C\right) \\ &= \frac{1-\delta^T}{1-\delta} v\left(p^P, q^P\right) + \frac{\delta^T}{1-\delta} v\left(p^C, q^C\right). \end{aligned} \tag{17}$$

[12] Because of the sequential nature of the stage game, the result of Abreu (1988) on optimal penal codes does not necessarily apply here, as explained by Mailath *et al.* (2008).

We also let V^{DP} be the present value of the regulator's future payoffs following a deviation from the punishment phase. Formally, it is given by

$$V^{DP} \equiv v\left(p^S, \widehat{q}\left(p^S\right)\right) + \delta V^P. \tag{18}$$

where, in writing the regulator's payoff following a deviation, we use the fact that the best possible price anticipating the firm's best reply is given by the solution to the static problem (3).

We are now in the position to check that, provided that she has no incentive to deviate from the subsequent punishment, the regulator has not incentive to deviate from the offered contract. To see this, write the regulator's incentive compatibility constraint

$$\frac{1}{1-\delta} v\left(p^C, q^C\right) \geq v\left(p^S, \widehat{q}\left(p^S\right)\right) + \delta V^P \tag{19}$$

which, using (17), becomes

$$\frac{1}{1-\delta} v\left(p^C, q^C\right) \geq v\left(p^S, \widehat{q}\left(p^S\right)\right) + \delta\left[\frac{1-\delta^T}{1-\delta} v\left(p^P, q^P\right) + \frac{\delta^T}{1-\delta} v\left(p^C, q^C\right)\right]. \tag{20}$$

Comparing period by period both sides of the inequality, it is easy to see that sufficient conditions for (20) to hold are that the offered contract delivers higher payoffs than the optimal static contract and the punishment contract, two conditions that must be clearly satisfied by any equilibrium contract. It remains to formalise the condition which ensures that the regulator has no incentive to deviate from the punishment phase. Formally, this is simply

$$V^P \geq V^{DP} \tag{21}$$

which, after using (17) and (18), becomes

$$\left(1-\delta^T\right) v\left(p^P, q^P\right) + \delta^T v\left(p^C, q^C\right) \geq v\left(p^S, \widehat{q}\left(p^S\right)\right). \tag{22}$$

Since $v\left(p^C, q^C\right)$ is always greater than $v\left(p^S, \widehat{q}\left(p^S\right)\right)$, this inequality is satisfied provided that $v\left(p^P, q^P\right)$ is not too small relatively to $v\left(p^S, \widehat{q}\left(p^S\right)\right)$ and/or δ^T is not too low.

Using the one-shot deviation principle (Mailath and Samuelson, 2006), it is easy to check that our pair of stick-and-carrot strategies forms a subgame perfect equilibrium of our game, provided that $\mathcal{F}^C$, $\mathcal{F}^P$ and T are chosen so that (15), (16) and (22) are satisfied.

Clearly, there are many combinations of p^C, q^C, p^P, q^P and T that support such an equilibrium. In principle, it could be possible to single out among them the combination which maximises the regulator's payoff by simply solving the problem of maximising the discounted present value of the regulator's payoff subject to constraints (15), (16) and (22). To our purposes, it is however sufficient to solve the simpler problem of finding the optimal price and quality pair for the "cooperative" phase, which we denote with p^* and q^*, and simply provide the conditions on p^P, q^P and T for the optimal p^* and q^* to be an equilibrium. This problem can be formally stated as follows

$$\max_{p^C, q^C} \sum_{t=0}^{\infty} \delta^t v\left(p^C, q^C\right) = \frac{1}{1-\delta} v\left(p^C, q^C\right) \tag{23}$$

$$\text{s.t. } \frac{1-\delta^{T+1}}{1-\delta} \pi\left(p^C, q^C\right) \geq \widehat{\pi}(p^C) + \delta \frac{1-\delta^T}{1-\delta} \pi\left(p^P, q^P\right) \tag{24}$$

$$\pi\left(p^P, q^P\right) \geq \frac{1}{1-\delta^T} \widehat{\pi}\left(p^P\right) - \frac{\delta^T}{1-\delta^T} \pi\left(p^C, q^C\right) \tag{25}$$

$$\left(1-\delta^T\right) v\left(p^P, q^P\right) + \delta^T v\left(p^C, q^C\right) \geq v\left(p^S, \widehat{q}\left(p^S\right)\right) \tag{26}$$

where we take p^P, q^P and T as given. In this problem, the first two constraints come simply from using in (15) and (16) the definitions of Π^D and Π^P given in (13) and (14). We can then state the main result of this Subsection:

Proposition 4 *When players use stick-and-carrot and $\delta \to 1$, the optimal contract satisfies the second best conditions $\frac{v_p(p^*, q^*)}{v_q(p^*, q^*)} = \frac{\pi_p(p^*, q^*)}{\pi_q(p^*, q^*)}$ and $\pi(p^*, q^*) = 0$, provided that $T \geq -\frac{\widehat{\pi}(p^*)}{\pi(p^P, q^P)}$ and $\pi\left(p^P, q^P\right) < 0$.*

Proof In solving the problem, we initially ignore constraint (26), which will be checked to hold once our solutions are found. The Lagrangean of the regulator's maximization problem is as follows

$$\begin{aligned} L \;=\; & \frac{1}{1-\delta} v\left(p^C, q^C\right) + \lambda \left[\frac{1-\delta^{T+1}}{1-\delta} \pi\left(p^C, q^C\right) - \widehat{\pi}(p^C) - \delta \frac{1-\delta^T}{1-\delta} \pi\left(p^P, q^P\right)\right] \\ & + \mu \left[\pi\left(p^P, q^P\right) - \frac{1}{1-\delta^T} \widehat{\pi}\left(p^P\right) + \frac{\delta^T}{1-\delta^T} \pi\left(p^C, q^C\right)\right]. \end{aligned}$$

FOCs are given by

$$L_p = \frac{1}{1-\delta} v_p(p^*, q^*) + \lambda \left[\frac{1-\delta^{T+1}}{1-\delta} \pi_p(p^*, q^*) - \widehat{\pi}_p(p^*) \right] + \mu \frac{\delta^T}{1-\delta^T} \pi_p(p^*, q^*) = 0; \tag{27}$$

$$L_q = \frac{1}{1-\delta} v_q(p^*, q^*) + \lambda \frac{1-\delta^{T+1}}{1-\delta} \pi_q(p^*, q^*) + \mu \frac{\delta^T}{1-\delta^T} \pi_q(p^*, q^*) = 0; \tag{28}$$

$$L_\lambda = \frac{1-\delta^{T+1}}{1-\delta} \pi(p^*, q^*) - \widehat{\pi}(p^*) - \delta \frac{1-\delta^T}{1-\delta} \pi(p^P, q^P) \geq 0; \quad \lambda \geq 0; \quad L_\lambda \cdot \lambda = 0; \tag{29}$$

$$L_\mu = \pi\left(p^P, q^P\right) - \frac{1}{1-\delta^T} \widehat{\pi}\left(p^P\right) + \frac{\delta^T}{1-\delta^T} \pi(p^*, q^*) \geq 0; \quad \mu \geq 0; \quad L_\mu \cdot \mu = 0. \tag{30}$$

Assume first that $\lambda = \mu = 0$. From (27) and (28), we have $v_p(p^*, q^*) = v_q(p^*, q^*) = 0$, clearly inconsistent with a second best situation.

Assume then $\lambda > 0$ and $\mu = 0$. Using these into (27) and (28), and rearranging, it gives

$$\frac{v_p(p^*, q^*)}{v_q(p^*, q^*)} = \frac{\pi_p(p^*, q^*)}{\pi_q(p^*, q^*)} - \frac{(1-\delta)}{(1-\delta^{T+1})} \frac{\widehat{\pi}_p(p^*)}{\pi_q(p^*, q^*)}. \tag{31}$$

We now take the limit for $\delta \to 1$ of this ratio and of (29) and (30) to get

$$\frac{v_p(p^*, q^*)}{v_q(p^*, q^*)} = \frac{\pi_p(p^*, q^*)}{\pi_q(p^*, q^*)} - \frac{1}{T+1} \frac{\widehat{\pi}_p(p^*)}{\pi_q(p^*, q^*)}; \tag{32}$$

$$\pi(p^*, q^*)(T+1) - \widehat{\pi}(p^*) - T\pi(p^P, q^P) = 0; \tag{33}$$

$$\pi(p^*, q^*) \geq \widehat{\pi}\left(p^P\right). \tag{34}$$

From (33) and (34) we get

$$T \leq \frac{\widehat{\pi}(p^*) - \widehat{\pi}\left(p^P\right)}{\widehat{\pi}\left(p^P\right) - \pi\left(p^P, q^P\right)}. \tag{35}$$

Notice that (32) illustrates that the Ramsey-like tangency condition (2) could only be obtained when $T = \infty$, something which however is not consistent with (35).

Assume now $\lambda > 0$ and $\mu > 0$. Using these into (27) and (28), and rearranging, it gives

$$\frac{v_p(p^*,q^*)}{v_q(p^*,q^*)} = \frac{\pi_p(p^*,q^*)}{\pi_q(p^*,q^*)} + \frac{\lambda(1-\delta)}{\lambda(1-\delta^{T+1}) + \mu\delta^T(1-\delta)} \frac{\widehat{\pi}_p(p^*)}{\pi_q(p^*,q^*)}. \quad (36)$$

We now take the limit for $\delta \to 1$ of this ratio and of (29) and (30) to get

$$\frac{v_p(p^*,q^*)}{v_q(p^*,q^*)} = \frac{\pi_p(p^*,q^*)}{\pi_q(p^*,q^*)} + \frac{\lambda}{\lambda(T+1)+\mu} \frac{\widehat{\pi}_p(p^*)}{\pi_q(p^*,q^*)}; \quad (37)$$

$$\pi(p^*,q^*)(T+1) - \widehat{\pi}(p^*) - T\pi\left(p^P,q^P\right) = 0; \quad (38)$$

$$\pi(p^*,q^*) = \widehat{\pi}\left(p^P\right). \quad (39)$$

From (38) and (39) we get

$$T = \frac{\widehat{\pi}(p^*) - \widehat{\pi}\left(p^P\right)}{\widehat{\pi}(p^P) - \pi\left(p^P,q^P\right)}. \quad (40)$$

Also in this case, (37) makes clear that, under these hypotheses on the multipliers, the Ramsey-like tangency condition (2) cannot be obtained, since (37) is equal to (2) only when $\lambda = 0$ which clearly contradicts the initial hypothesis on its value.

Finally, assume $\lambda = 0$ and $\mu > 0$. Using these into (27) and (28), and rearranging, it gives

$$\frac{v_p(p^*,q^*)}{v_q(p^*,q^*)} = \frac{\pi_p(p^*,q^*)}{\pi_q(p^*,q^*)}. \quad (41)$$

Also, taking the limit for $\delta \to 1$ of (29) and (30) we get

$$\pi(p^*,q^*)(T+1) - \widehat{\pi}(p^*) - T\pi(p^P,q^P) \geq 0; \quad (42)$$

$$\pi(p^*,q^*) = \widehat{\pi}\left(p^P\right). \quad (43)$$

From (42) and (43) we get

$$T \geq \frac{\widehat{\pi}(p^*) - \widehat{\pi}\left(p^P\right)}{\widehat{\pi}\left(p^P\right) - \pi\left(p^P,q^P\right)}. \quad (44)$$

In this case, $\pi(p^*,q^*) = 0$ implies that constraints (42) and (43) should be read as

$$T \geq -\frac{\widehat{\pi}(p^*)}{\pi\left(p^P,q^P\right)}; \quad (45)$$

$$0 = \widehat{\pi}\left(p^P\right); \quad (46)$$

since $\widehat{\pi}\left(p^P\right) > \pi\left(p^P,q^P\right)$ by definition, then constraint (46) implies that a positive T must be associated with $\pi\left(p^P,q^P\right) < 0$.

Despite the solutions in the different cases are not comparable with each other, they may be compared against the second best solution. Since contracts are stationary, there is no contract whose discounted present value is higher than the one that ensures in each period the static second-best. Hence, the unique solution to problem (23)-(26) is the one that obtains the static second-best, i.e. the one obtained when $\lambda = 0$ and $\mu > 0$.

Finally, notice that, when $\delta \to 1$, constraint (26) always holds. ■

The Proposition illustrates the features of the relational regulatory contract that, when δ tends to 1, maximises the regulator's objective function when players use stick-and-carrot strategies as described before. The restriction to the case when δ approaches 1 is legitimate for two different reasons. In the first place, the purpose of the Section is to show the effect of a different and more complex strategy (and a harsher punishment) on the impossibility to reach the second best even with an infinitely patient firm, which we obtained in the case of grim trigger strategies. Second, the result is more general than it seems; for reasons of continuity, there always exists a critical discount factor sufficiently close to 1, but strictly lower than 1, such that our contract exists for any discount factor greater than this critical level.

The Proposition illustrates that, when δ tends to 1, the optimal regulatory relational contract secures a tangency condition between the regulator's iso-welfare contours and the firm's isoprofit; in other words, the regulatory contract always ensures that the regulator and the firm face the same marginal rate of substitution between price and quality. As already discussed, this is one of the conditions that grants the socially optimality, *à la* Ramsey, of the regulatory contract.

Also, the optimal regulatory relational contracts grants the second condition for social optimality, namely zero profits to the firm when it adheres to the contract. We show that an appropriate choice of the punishment phase in case of deviation ensures this. Indeed, to this result, it is necessary not only a long enough punishment phase (i.e. T above a critical level), but also that the firm is inflicted negative profits during the punishment phase. This sheds light on one of our previous results which stated that, when the static equilibrium used to punish the deviating firm ensured it positive profits, the optimal contract chosen by a regulator using a grim trigger strategy could not ensure the second best even with an infinitely patient firm. We show here that even a more "sophisticated" strategy , like the stick-and-carrot adopted in this section by the regulator, is not enough *by itself* to ensure the second best when δ tends to 1; rather, it must also be the case that the regulator can punish the firm with negative profits in case of a deviation. Notice that this possibility is somehow in contrast with what it is normally posited in most of the regulation literature (even in dynamic games with a non-myopic firm, see De Fraja and Iozzi, 2008) where, in the light of the nature of the goods produced by the regulated firm, the regulator must make sure, even in ex-ante terms, that the firm obtains non-negative profits in each period.

4.2 Imperfectly observable quality

In this Subsection, we study the case of a regulator being unable to perfectly observe the quality choice of the firm. In previous Sections, quality was assumed to be perfectly observable, albeit uncontractible; we now assume that quality cannot be observed and that it is not always possible for the regulator to infer the firm's quality choice by observing the market demand. For ease of exposition, we concentrate in this Subsection on the case of a benevolent regulator, despite all our results hold true for more general regulator's preferences.

We assume that demand depends on a stochastic consumers' preference parameter, ω. In every period, ω can have two possible realisations, $\underline{\omega}$ and $\overline{\omega}$, which are i.i.d. over time. We let $Prob\{\omega = \underline{\omega}\} = \alpha$ and $Prob\{\omega = \overline{\omega}\} = 1-\alpha$, with these probabilities known by all parties.

The realised preference parameter, together with the quality level, determines in each period the nature of the demand function, which may be "high", $x^H(p,q;\omega)$, or "low", $x^L(p,q;\omega)$, where $x^H(p,q;\omega) > x^L(p,q;\omega)$ for any admissible price and quality pair. In particular, we assume that

$$x = \begin{cases} x^H(p,q;\omega) & \text{if} \quad \omega = \overline{\omega} \text{ and } q > q^- \\ x^L(p,q;\omega) & \text{if} \begin{cases} \omega = \overline{\omega} \text{ and } q = q^- \text{ or} \\ \omega = \underline{\omega} \text{ and } q \geq q^- \end{cases} \end{cases} \tag{47}$$

This aims at describing in a simple way the case in which the demand has a random component and this very feature makes it impossible to infer the (unobservable) quality choice of the firm by the simple observation of demand. In particular, notice that, while the occurrence of high demand implies that the firm has chosen a quality level higher than its minimum, low demand may occur either as a result of low quality or of the 'low' preference parameter $\underline{\omega}$. This is clearly an extreme simplification, which however grants full tractability of the model and allows us to focus directly on the features of the optimal contract with noisy observation of the firm's behaviour.

Ex-post profits are now denoted by $\pi^k(p,q;\omega) = x^k(p,q;\omega)p - c(x^k(p,q;\omega),q)$ for $k = L, H$; similarly, the regulator objective function is given by $v(x^k(p,q;\omega)) \equiv v^k(p,q;\omega)$. Both players are assumed to be risk-neutral.

As to the structure of the firm's profits, we make the following simplifying assumption:

Assumption 1 *For any admissible price p and any $\omega \in \{\underline{\omega}, \overline{\omega}\}$, the firm's payoffs are*

$$\pi^L(p, q^-; \omega) > \pi^k(p, q; \omega) \text{ for any } q \neq q^-. \tag{48}$$

This assumption means that, for any possible price level, the firm gets higher profits by reducing quality to its minimum level; in other words, $\widehat{q}(p; \omega) = q^- \equiv \widehat{q}$, for any possibile realisation of ω and any price level. This may be justified by the high cost of quality and/or by a relatively small increase in the consumers' demand due to an increase in the quality level.

Our repeated game with imperfect monitoring is given by an infinite repetition of the following stage game:

1. the regulator makes an offer $\mathcal{F} \equiv \{p', q'\}$ in which she asks the firm to produce a good of quality q' and sets the market price p' at which the good has to be sold;
2. the firm chooses whether or not to accept the contract: if the firm accepts the contract, the game proceeds to the following stage; otherwise, it does not produce, reservation payoffs $\tilde{\pi}$ and $\tilde{V}$ are collected and the game ends;[13]
3. the firm chooses the actual quality level q'';
4. nature chooses the parameter $\omega \in \{\underline{\omega}, \overline{\omega}\}$ and demand is realised;
5. the regulator and the firm observe and collect their own payoffs.

We first study the equilibrium of the static game. When the firm chooses its quality after the regulator's decision on the price, because of the previous assumption, it chooses $\widehat{q}$. In the previous stage, the regulator anticipates the firm's choice of the lowest quality level and its effect both on the firm's profits and the regulator's objective function, and solves

$$\max_p v^L(p, \widehat{q}) \tag{49}$$
$$\text{s.t. } \pi^L(p, \widehat{q}) \geq 0.$$

Since $v_p^L < 0$ by assumption, the price that solves this problem, denoted with p^S, is the lowest price that satisfy the non-negativity constraints; formally,

[13] As in the previous Sections, we assume that the regulator's offer is such that the firm undertakes production in each period, so that this stage of the game can be safely ignored in the characterisation of the equilibrium.

p^S is the smallest solution with respect to p of $\pi^L(p, \widehat{q}) = 0$. In other words, the price that maximises the payoff of a first-mover regulator always induces the firm to obtain zero profits. Had we illustrate this result with a figure similar to Figure 2, we would have to draw the locus aa' as a horizontal line. Since the iso-welfare contours are upward increasing, the price and quality combination which maximises social welfare is the one where this locus crosses the zero-profit contour.

We now turn to the analysis of the repeated game. Typically, when players are symmetric and the stage game is simultaneous, repeated games with imperfect monitoring are solved by using symmetric public perfect equilibrium (SPPE) as solution concept. A SPPE is a symmetric strategy profile in which, at any point in time, players do not rely on their private information but base their decisions on public history. When players use grim trigger strategies, the equilibrium is obtained by using the so-called bang-bang property, which ensures that it is possible to implement the optimal SPPE randomizing only between the two extremal points of the set of payoffs supported by SPPE. The equilibrium has then the following features: in the "cooperative" phase, both players coordinate on the "cooperative" action (which gives players the highest payoffs supported by SPPE) until an unfavourable realisation of the imperfect monitoring parameter occurs. Once this public event occurs, players coordinate on a public randomisation device and either jointly stay in the "cooperative" phase or enter the "punishment" phase, which gives players the lowest payoffs supported by SPPE (see, for instance, Amelio and Biancini, 2010).[14]

Unfortunately, due to the extensive form of the stage game, we cannot apply these techniques here. In the repeated game with imperfect observability described above, an equilibrium with, *mutatis mutandis*, the same features as the one described in the previous paragraph would not be subgame perfect. Indeed, in case of low demand, the one-shot deviation principle would be violated; the regulator would always have an incentive to "cheat" on the randomisation device and defer the punishment phase of one period. To avoid this and still obtain a result which has the flavour of the above literature, we therefore use the simplifying assumption that the regulator is ex-ante able to commit to a given strategy during the course of play. In the literature, this is sometimes rationalised as if the player(s) were substituted by an automaton, which intuitively consists of an output function that assigns a (possibly mixed)

[14] Here and in the rest of this Subsection, the term "punishment" is used in an improper way, since the "punishment" phase does not necessarily intends to punish a deviation but it is instead a change of state aimed at providing the right incentives to the players.

stage-game action to each state of the game (uniquely determined by the history of the game), and a transition function that determines the transitions across states as a function of the outcomes of the stage game.[15]

We concentrate on the equilibrium resulting from the adoption of the following strategies:

- **regulator**: the regulator/automaton begins the game by making to the firm an offer $\mathcal{F}^C$. In the following repetitions of the game, if demand has been "low", with probability β she starts offering her optimal static contract $\mathcal{F}^S$ forever after, while, with probability $1-\beta$, she continues to offer $\mathcal{F}^C$. If demand has instead been "high", with probability γ she starts offering her optimal static contract $\mathcal{F}^S$ forever after, while, with probability $1-\gamma$, she continues to offer $\mathcal{F}^C$;
- **firm**: the firm chooses the quality q^C as long as the history of the game is such that no price and quality different from p^C and q^C was previously chosen; otherwise, it reverts indefinitely to its equilibrium strategy in the stage game.

Then, β (respectively, γ) is the probability of starting a "punishment" phase, conditional on the realisation of "low" (respectively, "high") demand. In other words, the mixed strategy to which the regulator commits envisages that she chooses to begin a punishment phase with probabilities β and γ, depending on the realisation of demand.[16]

Before proceeding with the analysis of the game, we introduce the following definitions: $\bar{v}\left(p^C, q^C\right) \equiv \alpha v^L\left(p^C, q^C\right) + (1-\alpha)v^H\left(p^C, q^C\right)$ and

[15]There is large literature exploring the role of commitment in repeated games and its effect on the players' payoffs (Atakan and Ekmekci, 2012a and 2012b; Cripps and Thomas, 1995; García Jurado and González Díaz, 2005). This literature is mainly interested either in commitment emerging endogenously, for instance as a tool to establish a reputation, or in the exogenous mechanisms which allow a player to commit, for instance delegation. We do not pursue these lines of research and simply assume the ability of our first-mover player to commit. Along the lines illustrated above, this ability to commit may be motivated, for instance, by the regulator being an agent of a fully informed principal (the government) and/or having a very strict procedural code, or because of the regulator's attempt to build a reputation of being "tough".

[16]In the SPPE of simultaneous dynamic games of imperfect information, players take their actions coordinating on the outcome of a public randomisation device. Probabilities are exogenously chosen to ensure the best "collective" outcome of the game. Here instead, due to the extensive form of the constituent game, a public randomisation device is unnecessary and it is simply sufficient to assume that the first mover uses a mixed strategy, with endogenously determined probabilities, conditional on the history of the game. The second mover instead uses a pure strategy, being perfectly able to observe any past action.

$\bar{\pi}\left(p^C, q^C\right) \equiv \alpha\pi^L\left(p^C, q^C\right) + (1-\alpha)\pi^H\left(p^C, q^C\right)$. Also, to compact notation, we write $\widehat{\pi}^L(p) = \pi^L(p, \widehat{q})$, so that we can write $\bar{\pi}(p, \widehat{q}) \equiv \alpha\widehat{\pi}^L(p) + (1-\alpha)\widehat{\pi}^L(p) = \widehat{\pi}^L(p)$.

First of all, notice that, for the above strategies to be an equilibrium, we only need to make sure that profitable deviations *for the firm* do not exist. This is because, by assumption, the regulator/automaton will never deviate from the established course of action of her strategy. We then turn to write the firm's incentive compatibility constraint. To be able to write this constraint, we first concentrate on the expected current value of the firm's profits when it adheres to the contract offered by the regulator and, using Abreu *et al.* (1986, 1990), we write them as the following recursive expression

$$\Pi^C = \bar{\pi}\left(p^C, q^C\right) + \delta\left\{\alpha\left[\beta\Pi^P + (1-\beta)\Pi^C\right] + (1-\alpha)\left[\gamma\Pi^P + (1-\gamma)\Pi^C\right]\right\} \quad (50)$$

Solving it for Π^C and using the fact that $\Pi^P = \frac{1}{1-\delta}\pi^L\left(p^S, q^S\right) = 0$, one gets

$$\Pi^C = \frac{1}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]}\bar{\pi}\left(p^C, q^C\right). \quad (51)$$

When the firms instead deviates from the contract, the expected current value of its profits is given by the expression

$$\Pi^D = \widehat{\pi}^L(p) + \delta\left[\beta\Pi^P + (1-\beta)\Pi^C\right]. \quad (52)$$

Incentive compatibility requires that $\Pi^C \geq \Pi^D$. Using (51), (52) and $\Pi^P = 0$, after some manipulations, we rewrite the incentive compatibility constraint as follows

$$\frac{1}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]}\bar{\pi}\left(p^C, q^C\right) \geq \frac{1}{1-\delta(1-\beta)}\widehat{\pi}^L(p). \quad (53)$$

It is straightforward to check that, given the regulator's commitment, the pair of strategies described above are perfect equilibrium strategies, provided that the contract offered by the regulator satisfies the firm's incentive compatibility constraint (53). Since there are typically many such contracts, it is interesting to characterise the one that gives the regulator the highest utility. To find this, we first write the present value of the regulator's payoff when the firm adheres to the offered contract:

$$V^C = \bar{v}\left(p^C, q^C\right) + \delta\left\{\alpha\left[\beta V^P + (1-\beta)V^C\right] + (1-\alpha)\left[\gamma V^P + (1-\gamma)V^C\right]\right\}. \quad (54)$$

Solving it for V^C and using the fact that $V^P \equiv \frac{1}{1-\delta} v^L \left(p^S, q^S\right)$, one gets

$$\begin{aligned} V^C = & \frac{1}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]} \bar{v}\left(p^C, q^C\right) + \\ & + \frac{\delta}{1-\delta} \frac{\alpha\beta+(1-\alpha)\gamma}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]} v^L\left(p^S, q^S\right). \end{aligned} \tag{55}$$

Using these results, we can write the regulator's problem as follows:

$$\max_{p^C, q^C, \beta, \gamma} \; V^C(p^C, q^C, \beta, \gamma) \tag{56}$$

$$\text{s.t.} \frac{1}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]} \bar{\pi}\left(p^C, q^C\right) \geq \frac{1}{1-\delta(1-\beta)} \widehat{\pi}^L\left(p^C\right) \tag{57}$$

plus the natural constraints on β and γ to be such that $\beta, \gamma \in [0, 1]$.

From the solution of (56)-(57), next Proposition follows.

Proposition 5 *Let p^*, q^*, β^* and γ^* be the price/quality pair and the probabilities of triggering a punishment phase that maximise social welfare when the trigger strategies of the regulator/automaton and of the firm are an equilibrium of the dynamic game with imperfect information. Then, provided that $\alpha \leq \frac{\bar{\pi}(p^*, q^*)}{\widehat{\pi}^L(p^*)}$, the following hold*

$$\frac{\bar{v}_p\left(p^*, q^*\right)}{\bar{v}_q\left(p^*, q^*\right)} = \frac{\bar{\pi}_p\left(p^*, q^*\right) - \frac{\bar{\pi}(p^*, q^*)}{\widehat{\pi}^L(p^*)} \widehat{\pi}^L_p\left(p^*\right)}{\bar{\pi}_q\left(p^*, q^*\right)} \tag{58}$$

$$\beta^* = -\frac{1-\delta}{\delta} \frac{1 - \frac{\bar{\pi}(p^*, q^*)}{\widehat{\pi}^L(p^*)}}{\alpha - \frac{\bar{\pi}(p^*, q^*)}{\widehat{\pi}^L(p^*)}} \tag{59}$$

$$\gamma^* = 0 \tag{60}$$

Proof Using (55) and only the relevant constraints on β and γ, we write the Lagrangean of problem (56)-(57) as follows:

$$\begin{aligned} L = & \frac{1}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]} \bar{v}\left(p^C, q^C\right) \\ & + \frac{\delta}{1-\delta} \frac{\alpha\beta+(1-\alpha)\gamma}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]} v^L\left(p^S, q^S\right) \\ & + \lambda \left[\frac{1}{1-\delta[\alpha(1-\beta)+(1-\alpha)(1-\gamma)]} \bar{\pi}\left(p^C, q^C\right) - \frac{1}{1-\delta(1-\beta)} \widehat{\pi}^L\left(p^C\right) \right] \\ & + \mu\gamma. \end{aligned} \tag{61}$$

where λ is the multiplier associated with constraint (57) and μ is the one associated with the non-negativity constraint on γ. FOCs are given by

$$L_p = \frac{\bar{v}_p(p^*,q^*) + \lambda \bar{\pi}_p(p^*,q^*)}{1-\delta[\alpha(1-\beta^*)+(1-\alpha)(1-\gamma^*)]} - \lambda \frac{\widehat{\pi}_p^L(p^*)}{1-\delta(1-\beta^*)} = 0 \tag{62}$$

$$L_q = \frac{\bar{v}_q(p^*,q^*) + \lambda \bar{\pi}_q(p^*,q^*)}{1-\delta[\alpha(1-\beta^*)+(1-\alpha)(1-\gamma^*)]} = 0 \tag{63}$$

$$L_\beta = \frac{\alpha\delta\left[\bar{v}(p^*,q^*) - v^L\left(p^S,q^S\right) + \lambda\bar{\pi}(p^*,q^*)\right]}{(1-\delta[\alpha(1-\beta^*)+(1-\alpha)(1-\gamma^*)])^2} - \lambda\frac{\delta\widehat{\pi}^L(p^*)}{[1-\delta(1-\beta^*)]^2} = 0; \tag{64}$$

$$L_\gamma = \frac{\delta(1-\alpha)\left[\bar{v}(p^*,q^*) - v^L\left(p^S,q^S\right) + \lambda\bar{\pi}(p^*,q^*)\right]}{(1-\delta[\alpha(1-\beta^*)+(1-\alpha)(1-\gamma^*)])^2} - \mu = 0; \tag{65}$$

$$L_\lambda = \frac{\bar{\pi}(p^*,q^*)}{1-\delta[\alpha(1-\beta^*)+(1-\alpha)(1-\gamma^*)]} - \frac{\widehat{\pi}^L(p^*)}{1-\delta(1-\beta^*)} \geq 0;$$
$$\lambda \geq 0; \qquad L_\lambda\lambda = 0; \tag{66}$$

$$L_\mu = \gamma \geq 0; \qquad \mu \geq 0; \qquad L_\mu\mu = 0. \tag{67}$$

From (62) and (63), it follows that $\lambda > 0$; if this were not the case, we would have that $\bar{v}_p(p^*,q^*) = \bar{v}_q(p^*,q^*) = 0$, which clearly contradicts the hypothesis that the (ex-ante) first best is out of reach. Therefore, $L_\lambda = 0$ in (66).

From (65), since $\bar{v}(p^*,q^*) > v^L\left(p^S,q^S\right)$, it follows that $\mu > 0$. From (67), this implies (60). Using this into $L_\lambda = 0$, one obtains $\frac{1}{1-\delta(1-\alpha\beta^*)}\bar{\pi}(p^*,q^*) = \frac{1}{1-\delta(1-\beta^*)}\widehat{\pi}^L(p^*)$. Solving it w.r.to β gives (59); since $\bar{\pi}(p^*,q^*) < \widehat{\pi}^L(p^*)$, this is always different from 0 provided that δ is different from 1. Also, $\alpha < \frac{\bar{\pi}(p^*,q^*)}{\widehat{\pi}^L(p^*)}$ ensures that $\beta^* > 0$.

Finally, using $\gamma^* = 0$ into (62) and (63), these can be rewritten as

$$\frac{1}{1-\delta(1-\alpha\beta^*)}\bar{v}_p(p^*,q^*) + \lambda\left[\frac{1}{1-\delta(1-\alpha\beta^*)}\bar{\pi}_p(p^*,q^*) - \frac{1}{1-\delta(1-\beta^*)}\widehat{\pi}^L(p^*)\right] = 0 \tag{68}$$

$$\frac{1}{1-\delta(1-\alpha\beta^*)}\bar{v}_q(p^*,q^*) + \lambda\frac{1}{1-\delta(1-\alpha\beta^*)}\bar{\pi}_q(p^*,q^*) = 0 \tag{69}$$

from which, after a simple rearranging and using (59), we obtain (58). ■

Proposition 5 allows us to characterise the regulator's optimal strategy. The regulator begins by offering a contract $\{p^*,q^*\}$. If the realisation of demand is "high", this necessarily implies that the firm has delivered the required quality level and then the regulator does not trigger the punishment phase. When the realization of demand is "low", the regulator switches to the static optimal contract $\left\{p^S, q^S\right\}$ with probability β^*.

This equilibrium never exists when the probability of realisation of the less favourable demand parameter is "too" high. The reason for this is clear when one looks at the extreme case of $\alpha = 1$; using this into the firm's incentive compatibility constraint (57), this reduces to $\pi^L\left(p^C, q^C\right) \geq \widehat{\pi}^L\left(p^C\right)$, which is clearly never satisfied. Indeed, when the probability of an unfavourable realisation of demand is high, the probability that the regulator will start the punishment phase is in turn high, so that the firm does not have an incentive to adhere to the offered contract. The threshold value of α is $\frac{\bar{\pi}(p^*,q^*)}{\widehat{\pi}^L(p^*)}$. This ratio, which plays a fundamental role in the entire Proposition, is clearly always less than one and gives an inverse measure of the static loss the firm faces when adhering to the offered contract: when $\frac{\bar{\pi}(p^*,q^*)}{\widehat{\pi}^L(p^*)}$ is close to zero, the difference between the cooperative and the deviation profits is very large and, on the other hand, the closer to 1 is the ratio, the smaller are the profits the firms has to give up when adhering to the contract.

The probability of triggering the punishment phase, β^*, provides the firm the appropriate incentives to offer the contractual quality. The optimal value of β^* also depends on the ratio $\frac{\bar{\pi}(p^*,q^*)}{\widehat{\pi}^L(p^*)}$. The larger is the loss from cooperation (i.e., the smaller is the above ratio), the greater will be the incentive to deviate from the offered contract, and therefore, the larger is the probability of triggering the punishment phase when demand is low. A similar (but clearly opposite) argument holds for the inverse relationship between δ and β^*. In the limiting case of an infinitely patient firm, β^* loses its incentive role and goes down to zero; on the other hand, the more impatient is the firm, the greater is the need to discipline it by a sufficiently large probability of "punishment"in case of low demand.

Finally, (58) shows that the contract $\{p^*, q^*\}$ is typically unable to ensure the (ex-ante) second best, the distortion depending again on the ratio $\frac{\bar{\pi}(p^*,q^*)}{\widehat{\pi}^L(p^*)}$. It is only in the limiting case of $\delta \to 1$ that the second best is achieved. Indeed, when δ tends to 1, β^* tends to 0 as well; taking the limit of both sides of the (57), it results that this constraint is always satisfied. It is then natural for the regulator to choose $\bar{\pi}\left(p^*, q^*\right) = 0$ to minimise the firm's rent. This in turn ensures that, in (58), the second best is achieved. It is however interesting to note that the condition on α necessary for the existence of the equilibrium constrains α to be equal to zero. In other words, to achieve the second-best, it is not only necessary that the firm is infinitely patient, but also that the realisation of demand parameter is always high.

Leaving aside these extreme parametric conditions, it is interesting to note that, differently from the other settings analysed in the paper, the distortion from the second best illustrated in (58) does not directly depend on the discount factor. This is because, in the present setting, the appropriate incentives to offer the contractual quality are mostly given by β^*, which indeed varies with δ.

4.3 Two-dimensional quality

In this Subsection we assume that the good has two different quality dimensions, q and z, both fully observable by all parties. While q remains, as before, not verifiable, z is verifiable. Demand is now given by $x(p, q, z)$, while the firm's cost function is $c(x, q, z)$. The regulator's objective function is $v(p, q, z)$. Apart from verifiability, all hypotheses on q are also valid for z.

The regulator's offer in Stage 1 is given by $\mathcal{F} \equiv \{p', q', z'\}$. In Stage 3, the firm chooses the quality levels q'' and z''; at the end of this stage the regulator observes both quality dimensions and payoffs are collected. Quality z is verifiable in the sense that the regulator could impose a directly enforceable penalty on the firm when she observes $z' \neq z''$; we assume this penalty to be arbitrarily large, so that the firm never has an incentive to do so.

Let $\widehat{\pi}(p, z) \equiv \pi\left(p, \widehat{q}(p, z), z\right)$, where $\widehat{q}(p, z)$ denotes the firm's optimal choice of q, for any given p and z. We also let p^S and z^S denote the price and (verifiable) quality offered by the regulator in equilibrium in the stage game. Finally, we denote with p^R, q^R and z^R the triplet of price and quality dimensions which solves the (static) problem of maximising social welfare subject to a non-negativity constraint on the firm's profits. At p^R, q^R and z^R, the following hold:

$$\frac{v_p\left(p^R, q^R, z^R\right)}{v_q\left(p^R, q^R, z^R\right)} = \frac{\pi_p\left(p^R, q^R, z^R\right)}{\pi_q\left(p^R, q^R, z^R\right)} \tag{70}$$

$$\frac{v_z\left(p^R, q^R, z^R\right)}{v_q\left(p^R, q^R, z^R\right)} = \frac{\pi_z\left(p^R, q^R, z^R\right)}{\pi_q\left(p^R, q^R, z^R\right)} \tag{71}$$

The structure of both the static and the dynamic game is very similar to the case of quality being mono-dimensional and its full description is omitted. Among the many possible self-enforcing regulatory relational contracts, denoted as before as $\mathcal{F}^C \equiv \left\{p^C, q^C, z^C\right\}$, we simply characterise here the structure of the optimal contract, $\mathcal{F}^* \equiv \{p^*, q^*, z^*\}$, which gives the highest payoff to the regulator when players use grim trigger strategies. This comes as the solution of the following problem:

$$\max_{p^C,q^C,z^C} \frac{1}{1-\delta} v\left(p^C, q^C, z^C\right) \tag{72}$$

$$\text{s.t.} \quad \frac{1}{1-\delta}\pi\left(p^C, q^C, z^C\right) \geq \widehat{\pi}(p,z) + \frac{\delta}{1-\delta}\widehat{\pi}(p^S, z^S) \tag{73}$$

We can then state the following Proposition:

Proposition 6 *When players use grim trigger strategies and quality has two dimensions, the price and quality combination* $\{p^*, q^*, z^*\}$ *solving problem (72)-(73) has the following properties:*

- *it satisfies the following conditions:*

$$\frac{v_p(p^*,q^*,z^*)}{v_q(p^*,q^*,z^*)} = \frac{\pi_p(p^*,q^*,z^*) - (1-\delta)\widehat{\pi}_p(p^*,z^*)}{\pi_q(p^*,q^*,z^*)} \tag{74}$$

$$\frac{v_z(p^*,q^*,z^*)}{v_q(p^*,q^*,z^*)} = \frac{\pi_z(p^*,q^*,z^*) - (1-\delta)\widehat{\pi}_z(p^*,z^*)}{\pi_q(p^*,q^*,z^*)} \tag{75}$$

  and

$$\pi(p^*,q^*,z^*) = (1-\delta)\widehat{\pi}(p^*,z^*) + \delta\widehat{\pi}(p^S,z^S); \tag{76}$$

- *the optimal unverifiable quality* q^* *is always set to satisfy the static quality-adjusted Ramsey condition, given the optimal price and verifiable quality.*

The proofs of this and next Proposition are very similar to the ones of Proposition 2 and 3 and are therefore omitted. Conditions (74), (75) and (76) are also very similar to the ones presented in Proposition 2, when quality was mono-dimensional and entirely unverifiable, and most of the discussion provided there applies here too. The main result of this analysis is instead the last part of Proposition 6, which confirms the dichotomy result we obtain in the previous section: the optimal relational contract create a distortion only on the verifiable components of the firm's output. Indeed, we find that both the price and the verifiable quality dimensions are distorted away from the second best to give the firm enough incentives to adhere to the contact, while the unverifiable component of quality is set to the second-best level (given the level of the other components of the output). Notice that, since this result only depends on the demand and cost first derivatives, it does not depend on the complementarity or substitability relationship between the two quality component, which typically affect the second order demand and cost derivatives.

We also consider the case of consumers with quasi-linear preferences and of an utilitarian regulator, as in Section 3.3. In a perfect analogy with the case of quality being monodimensional, next Proposition presents, for the case of multidimensional quality, what we have already defined as a formula for relational Ramsey pricing:

Proposition 7 *Assume consumers have quasi-linear preferences and the regulator is a benevolent utilitarian. When players use grim trigger strategies and quality has two dimensions, the optimal contractual price satisfies*

$$\frac{p^* - c_x(x(p^*, q^*, z^*), q^*, z^*)}{p^*} = \frac{1}{\eta(p^*, q^*, z^*)} \left[\frac{\lambda - 1}{\lambda} - (1 - \delta) \frac{\widehat{\pi}_p(p^*, z^*)}{x(p^*, q^*, z^*)} \right].$$

5 Conclusions

This paper tackles the issue of how regulatory authorities might induce a regulated firm to deliver unverifiable (then ex ante not contractible) quality. Although the recent literature has successfully showed that enforcing a socially optimal level of quality is possible by incorporating some sophisticated instruments, we show that the main problem of unverifiability could be eased by a standard (easy to apply) fixed-price contract. Our paper defines the optimal fixed-price contract the regulator needs to offer a regulated firm when the quality is endogenous, observable but not verifiable. We suggest that, using the discretionary powers of the regulator and exploiting the repeated nature of the interaction between the regulator and the firm, there exist self-enforcing relational agreements which may help overcoming the difficulties due to the unverifiable nature of quality. We show that, in an infinitely repeated contractual relation, if the regulator rewards the firm by means of a high regulated price when it delivers a mandated quality level and punishes it by reducing the price in future periods when it deviates from such a level, the optimal relational contract improves upon the static equilibrium level. This contract however typically entails distortions from the quality and price of second best, unless the punishment is so harsh to induce zero or negative profit. Adding a further source of inefficiency such as the impossibility for the regulator to perfectly monitor the firm's quality choices makes the second best price/quality pair out of reach even with an infinitely patient firm.

References

Abreu, D., D. Pearce and E. Stacchetti (1986). "Optimal Cartel Equilibria with Imperfect Monitoring". *Journal of Economic Theory*, 39: 251-269.

Abreu, D. (1988). "On the Theory of Infinitely Repeated Games with Discounting". *Econometrica*, 56: 383-948.

Abreu, D. (1986). "External Equilibria of Oligopolistic Supergames". *Journal of Economic Theory*, 39: 191-225.

Abreu, D. (1988). "On the Theory of Infinitely Repeated Games with Discounting". *Econometrica*, 56: 383-948.

Abreu, D., P. K. Dutta and L. Smith (1994). "The Folk Theorem for Repeated Games: a NEU Condition". *Econometrica*, 62: 934-948.

Albano, G. L., B. Cesi and A. Iozzi (2011). *"Relational" Procurement Contracts: A Simple Model of Reputation Mechanism.* CEIS Working Paper No. 209.

Amelio, A. and S. Biancini (2010). "Alternating Monopoly and Tacit Collusion". *Journal of Industrial Economics*, 58(2): 402-423.

Armstrong, M., S. Cowan and J. Vickers (1994). *Regulatory Reform: Economic Analysis and British Experience*. Cambridge (MA): MIT Press.

Armstrong, M., R. Rees and J. Vickers (1995). "Optimal Regulatory Lag under Price Cap Regulation". *Revista Espanola de Economia*, 12: 93-116.

Atakan, A. E. and M. Ekmekci (2012a). "Reputation in long-run relationships". *Review of Economic Studies*, forthcoming.

Atakan, A. E. and M. Ekmekci (2012b). "Reputation in the long-run with imperfect monitoring". mimeo.

Auray, S., T. Mariotti and F. Moizeau (2011). "Dynamic Regulation of Quality". *RAND Journal of Economics*, 42(2): 246-265.

Baumol W. J. and D. F. Bradford (1970). "Optimal Departures from Marginal Cost Pricing". *American Economic Review*, 60(3): 265-283.

Beesley, M. E. and S. C. Littlechild (1989). "The Regulation of Privatized Monopolies in the United Kingdom". *RAND Journal of Economics*, 20(3): 454-472.

Bös, D. (1981). "Economic Theory of Public Enterprise". Berlin: Springer-Verlag.

Calzolari G. and G. Spagnolo (2009). *Relational Contracts and Competitive Screening*. CEPR Discussion Paper no. 7434.

Cowan, S. (2002). "Price-cap regulation". *Swedish Economic Policy Review*, 9: 167-188.

Cripps, M. W., and J. P. Thomas (1995). "Reputation and Commitment in Two-Person Repeated Games without Discounting". *Econometrica*, 63: 1401-1420.

Currier, K. M. (2007). "Quality-corrected price caps". *Bulletin of Economic Research*, 59(3): 255-268.

Dalen D. M. (1997). "Regulation of Quality and the Ratchet Effect: Does Unverifiability Hurt the Regulator?". *Journal of Regulatory Economics*, 11(2): 139-155.

De Fraja G. and A. Iozzi (2008). "The quest for quality: a quality adjusted dynamic regulatory mechanism". *Journal of Economics & Management Strategy*, 17 (4): 1011-1040.

Doni N. (2006). "The Importance of the Reputation in Awarding Public Contracts". *Annals of Public and Cooperative Economics*, 77: 401-429.

Feldstein, M. (1972). "Distributional Equity and the Optimal Structure of Public Prices". *American Economic Review*, 62(1), 32–36. (corrected version of p. 33, footnote 7: *American Economic Review*, 62: 763).

Fudenberg, D. and E. Maskin (1986). "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information". *Econometrica*, 54: 533-554.

García Jurado, J. and I. González Díaz (2006). "The Role of Commitment in Repeated Games". *Optimization*, 55: 541-553.

Hviid, M. (2000). "Long-Term Contracts and Relational Contracts". In B. Bouckaert and G. De Geest (eds.), *Encyclopedia of Law and Economics*, 46-72. Cheltenham: Edward Elgar.

Iossa, E. and P. Rey (2010). *Building Reputation for Contract Renewal: Implications for Performance Dynamics and Contract Duration*. CEIS Working Paper No. 155.

Iozzi A., J. A. Poritz and E. Valentini (2002). "Social Preferences and Price Cap Regulation". *Journal of Public Economic Theory*, 4(1): 95-114.

Kim I. G. (1998). "A model of selective tendering: does bidding competition deter opportunism by contractor?". *The Quarterly Review of Economics and Finance*, 38: 907-925.

Kremer, I., Z. Wiener and E. Winter (2006). "Optimal Recurrence in Dynamic Auctions". mimeo.

Laffont J.-J. and J. Tirole (1988). "Repeated Auctions of Incentive Contracts, Investment, and Bidding Parity with an Application to Takeovers". *RAND Journal of Economics*, 19(4): 516-537.

Laffont J.-J. and J. Tirole (1993). *A Theory of Incentive in Procurement and Regulation*. Cambridge (MA.): MIT Press.

Levin J. (2003). "Relational Incentive Contracts". *American Economic Review*, 93 (3): 835-857.

Lewis T. and D. E. M. Sappington (1992). "Incentives for Conservation and Quality-Improvement by Public Utilities". *American Economic Review*, 82(5): 1321-1340.

MacLeod, W. B. (2007). "Reputations, Relationships, and Contract Enforcement". *Journal of Economic Literature*, 45(3): 595-628.

Mailath, G. J., V. Nocke, and L. White (2008). "When the Punishment Must Fit the Crime: Remarks on the Failure of Simple Penal Codes in Extensive Form Games". Mimeo.

Mailath, G. J. and L. Samuelson (2006). *Repeated Games and Reputations: Long Run Relationships*. Oxford University Press

National Audit Office (2010). "Review of the UK's Competition Landscape". London: HMSO.

Oates, W. E. and P. R. Portney (2003). "The political economy of environmental policy". In K.-G. Mäler and J. Vincent (eds.), *Handbook of Environmental Economics*. North-Holland/Elsevier Science: Amsterdam.

Sappington D. E. M. (2005). "Regulating service quality: A survey". *Journal of Regulatory Economics*, (2): 123-154.

Sasaki, D. and R. Strausz (2008). "License length, renewal, and collusion". mimeo.

Sheshinski E. (1976). "Price, Quality and Quantity Regulation in Monopoly Situations". *Economica*, 43: 127-137.

Sorin, S. (1995). "A Note on Repeated Extensive Games". *Games and Economic Behavior*, 9: 116-123.

Spagnolo G. (2012). "Reputation, competition, and entry in procurement ". *International Journal of Industrial Organization*, 30: 291-296.

Spence M. (1975). "Monopoly, Quality and Regulation". *Bell Journal of Economics*, 6: 417-429.

Stigler, G. J. (1971). "The Theory of Economic Regulation". *Bell Journal of Economics*, 2: 3-21.

Valentini, E. (2006). "Uncovering Regulators' (Implicit) Social Welfare Weights under Price Cap Regulation". *Economics Letters*, 90 (1): 1-5.

Waddams Price C., B. Brigham and L. Fitzgerald (2008). "Service Quality in Regulated Monopolies". *Annals of Public and Cooperative Economics*, 79(2): 197-225.

Weisman, D.L. (2005), "Price Regulation and Quality". *Information Economics and Policy*, 17: 165-174.

Wen, Q. (1994). "The Folk Theorem for Repeated Games with Complete Information". *Econometrica*, 62: 949-954.

Wen, Q. (2002). "A Folk Theorem for Repeated Sequential Games". *Review of Economic Studies*, 69(2): 493-512.